

Exercise Sheet 3

winter term 2026/27

Discussion on 02.11.2026

Exercise 1. Check the conditions of Theorem 2.24 from the lecture for the following Runge-Kutta methods:

- Euler-Collatz method
- implicit trapezoidal method
- Heun's method
- classical Runge-Kutta method

Exercise 2. Check the conditions of Theorem 3.25 from the lecture for the following Runge-Kutta methods:

- Euler-Collatz method
- implicit trapezoidal method
- Heun's method
- classical Runge-Kutta method

Exercise 3. Determine the consistency order of the leapfrog and the Milne-Simpson method.

Exercise 4 (Programming exercise). 1. Implement the Euler-Collatz method, the implicit trapezoidal method, and the classical Runge-Kutta method. You can use the function `fixpoint_iteration` from `ODE_solver.jl` to solve the non-linear system of equations.

2. Implement a routine that computes the ℓ^∞ error of $(y_k)_{k=1,\dots,K}$ and $(y(t_k))_{k=1,\dots,K}$, if the exact solution is known.
3. Compare the methods to the explicit Euler-method, the implicit Euler-method and the implicit midpoint method.