Exercise Sheet 4

winter term 2025/26

Discussion on 17.11.2025

Exercise 1. Determine the coefficients of the Adams-Moulton-method for $m = 3$.

Exercise 2. Show that the Adams-Bashforth-method with m steps is consistent of order m and the Adams-Moulton-method with m steps is consistent of order $m+1$.

Exercise 3. Let $(\hat{\alpha}_\ell, \hat{\beta}_\ell)_{\ell=0,\dots,m}$ define a linear, explicit multistep method and $(\alpha_\ell, \beta_\ell)_{\ell=0,\dots,m}$ define a linear, implicit multistep method. The approximation y_{k+m} is defined by $y_{k+m} = y_{k+m}^{(j)}$, where $y_{k+m}^{(j)}$ is defined by the following algorithm:

1. Compute the solution of the explicit method, i.e., compute

$$\tilde{y}_{k+m} = -\sum_{\ell=0}^{m-1} \hat{\alpha}_\ell y_{k+\ell} + \tau \sum_{\ell=0}^{m-1} \hat{\beta}_\ell f(t_{k+\ell}, y_{k+\ell}).$$

2. Set $y_{k+m}^{(0)} = \tilde{y}_{k+m}$.

3. Compute j steps of the fixpoint iteration of the implicit method, i.e., compute $y_{k+m}^{(j)}$ with

$$y_{k+m}^{(i+1)} = -\sum_{\ell=0}^{m-1} \alpha_\ell y_{k+\ell} + \tau \sum_{\ell=0}^{m-1} \beta_\ell f(t_{k+\ell}, y_{k+\ell}) + \tau \beta_m f(t_{k+m}, y_{k+m}^{(i)}).$$

Show that, if f is Lipschitz continuous in the second argument, this algorithm defines an explicit multistep method with order of consistency $p = \min\{p_{\text{expl}} + j, p_{\text{impl}}\}$, where p_{expl} und p_{impl} are the orders of the explicit resp. the implicit method.

Exercise 4 (Programming exercise)**.** Implement the Adams-Bashforth method and the Adams-Moulton method with 3 steps and compare the results to those of the already implemented methods. For the first two steps, the midpoint method can be employed. Do the methods converge with the optimal rates?