Topics

1. ### Interacting Markov chains

   Definition of Markov chains and basic properties. Recurrent construction of an interacting Markov chain on a finite state space

2. ### Proof of rate of convergence in ergodic theorem via interacting Markov chains

   Definition and existance of invariant measure for Markov chains defined on a finite state spaces. Using the constructed interacting Markov chain from the previous talk, prove the convergence of values of a Markov chain to its invariant measure as time goes to infinity.

3. ### Wasserstein distance and space of probability measures. Definition and basic properties

   Definition of Wasserstein distance on the space of probability measures with finite p-th moment. Proof of the fact that the Wasserstein space is a complete separable metric space.

4. ### Wasserstein distance and space of probability measures. Connection with weak convergence

   Definition of weak convergence of probability measures. Connection between weak convergence and convergence in Wasserstein distance.

5. ### Examples of Markov chains with interaction

   Discussion of examples of Markov chains with interaction, where the motion of particles depends on their masses. It will give us some intuition needed for understanding of interacting dynamics in more complicated cases later.

6. ### Continuous stochastic processes as random element in the space of continuous functions

   Definition of stochastic processes and stochastic processes with continuous paths. Definition of random elements in metric spaces. Proof that any continuous process is a random element in a space of continuous functions equipped with uniform distance. This discussion will be needed for understanding of the next talk.

7. ### Random measures

   Definition of random measures. Proof that every random measure with finite p-th moment is a random element of the corresponding Wasserstein space.

8. ### Different types of convergence and Skorokhod theorem

   Remind the definitions of convergence almost surely and weak convergence. Prove the Skorokhod theorem on the relationship between weak convergence and convergence almost surely. This statement will be needed for a proof presented in the next talk

9. ### Stochastic kernels on spaces of measures

   Definition of stochastic kernels on spaces of measures. Examples. Idea of proof of the statement about existence of a stochastic kernel with given finite dimensional distributions

10. ### Filtration, continuous martingales and Brownian motion

    Definitions of filtration, continuous martingale and Brownian motion. Proof of the Burkholder-Davis-Gundy inequality.

11. ### Stochastic integral

    Construction of a stochastic integral with respect to a Brownian motion

12. Ito formula and Levi characterization of Brownian motion

    Proof of Ito formula and Levi characterization of Brownian motion

13. Stochastic differential equations and stochastic flows

    Consider an SDE driven by an infinite family of Brownian motions. Definition of a strong solution to such an SDE. Discuss existence and uniqueness of strong solutions. Formulate a result about existence of stochastic flows

14. Deterministic differential equations with interaction

    Consider some examples of deterministic differential equations with interaction. This will be the first look on equations with interaction in continuous time but only in the deterministic case.

15. Brownian sheet and stochastic differential equations with interaction

    Definition of Brownian sheet. Shortly repeat the introduction of stochastic integral but now for a Brownian sheet. Proof of Ito formula for a Brownian sheet, using the usual Ito formula. Introduce equations with interaction and give the definition of strong solutions

16. Existence of solutions for SDE with interaction.

    Proof of the theorem about existence and uniqueness of solutions using the method of iterations

17. Evolutionary measure-valued processes on discrete spaces

    Definition of Markov evolutionary measure-valued processes. Examples of Evolutionary measure-valued processes on discrete spaces

18. Stochastic flows with interaction and evolutionary processes

    Proof of Markov property of a solution to an SDE with interaction. Proof of the statement that a solution to an SDE with interaction is an evolutionary process

19. Evolutionary processes generated by stochastic semigroups

    Definition of stochastic semigroup. Examples of some evolutionary processes generated by a stochastic semigroup

20. Existence of stationary solutions of SDE with interaction

    Definition of solutions to SDEs with interaction without initial conditions. Proof of the statement about the existence and uniqueness of a stationary solutions to such an equation

21. Tightness and weak compactness

    Definition of tightness and weak compactness. Prokhorov theorem about relationship between tightness and weak compactness

22. Characterization of weak compactness for measure-valued processes

    Discuss the compactness of continuous processes. In particular present the statement about compactness of measure-valued processes.

23. Weak limit of processes with interaction

    The weak convergence of solutions to an SDE with interaction under some rescaling its coefficients. This will lead us to an interacting particles system called the Arratia flow or coalescing Brownian particles

24. Arratia flow

    Introduce Arratia flow using a system of independent Brownian motions and discuss some its properties.