UNIVERSITÄT LEIPZIG

Problem sheet 5

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Solutions will be collected during the lecture on Monday May 13.

1. **[4 points]** Using Cramer's rule, solve the following system of linear equations:

$$\begin{cases} 2x_1 + 2x_2 - x_3 + x_4 &= 4, \\ 4x_1 + 3x_2 - x_3 + 2x_4 &= 6, \\ 8x_1 + 5x_2 - 3x_3 + 4x_4 &= 12, \\ 3x_1 + 3x_2 - 2x_3 + 2x_4 &= 6. \end{cases}$$

2. **[2+3 points]** Find the inverse matrices for

$$a) \begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}; \quad b) \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}.$$

via the computation of $i-j$ cofactors.

3. **[2 points]** Let e_1, e_2, e_3 be a standard basis of $\mathbb{R}^3$ and $x = (6, 2, -7)_e$. Show that

$$e_1' = (2, 1, -3), \quad e_2' = (3, 2, -5), \quad e_3' = (1, -1, 1)$$

if a basis and find the coordinates of x in this basis.

4. **[3 points]** Find the change-of-basis matrix from the basis

$$e_1 = (1, 2, 1), \quad e_2 = (2, 3, 3), \quad e_3 = (3, 7, 1)$$

to

$$e_1' = (3, 1, 4), \quad e_2' = (5, 2, 1), \quad e_3' = (1, 1, -6).$$

5. **[3 points]** Let T be a linear map from V to W such that its matrix in the bases e_1, e_2, e_3 of V and $\varepsilon_1, \varepsilon_2$ of W is

$$\begin{pmatrix} 0 & -1 & 2 \\ 3 & -4 & 5 \end{pmatrix}.$$

Find the matrix of T in the bases $e_1' = e_1, \quad e_2' = e_1 + e_2, \quad e_3' = e_1 + e_2 - e_3$ of V and $\varepsilon_1' = \varepsilon_1, \quad \varepsilon_2' = \varepsilon_1 - \varepsilon_2$ of W.

6. **[4 points]** Let T be a linear map from $\mathbb{R}_3[z]$ to $\mathbb{R}_2[z]$ defined as $(Tp)(z) = p'(z)$. Find the matrix of T in the basis:
a) $p_0(z) = 1, \quad p_1(z) = z, \quad p_2(z) = z^2, \quad p_3(z) = z^3$ in $\mathbb{R}_3[z]$ and $r_0(z) = 1, \quad r_1(z) = z, \quad r_2(z) = z^2$ in $\mathbb{R}_2[z]$;
b) $q_0(z) = 1, \quad q_1(z) = z - t, \quad q_2(z) = (z-t)^2, \quad q_3(z) = (z-t)^3$ in $\mathbb{R}_3[z]$ and $l_0(z) = 1, \quad l_1(z) = z - s, \quad l_2(z) = (z-s)^2$ in $\mathbb{R}_2[z]$, where t and s are real numbers.
Find coordinates of Tp in the basis l_0, l_1, l_2 (if p is written in the basis $q_0, \ldots, q_3$).

7. **[3 points]** Let T be a linear transformation in $\mathbb{R}^2$ with the matrix $\begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix}$ in the basis $e_1 = (1, 2), \quad e_2 = (2, 3)$ and let S be a linear transformation in $\mathbb{R}^2$ with the matrix $\begin{pmatrix} 4 & 6 \\ 6 & 9 \end{pmatrix}$ in the basis $e_1' = (3, 1), \quad e_2' = (4, 2)$. Find the matrix of $T + S$ in the basis e_1', e_2'.