The system of sticking diffusion particles with variable weights

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19.06.2009

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Interacting particles with mass changing and sticking

Main object

Infinite system of diffusion particles on the real line such that

1. start with some set of points with masses
2. independent motion up to the moment of the meeting
3. sticking
4. adding of the mass under sticking
5. diffusion changes correspondingly to the changing of the mass

Some history of the system of diffusion particles with interaction

- $\star$ diffusion particles with sticking with out mass
  - R.W. Darling (1987) *Constructing nonhomeomorphing stohastic flows,* Text. Article, Mem. Am. Math. Soc. 376, 97 p.
  - R.A. Arratia (1979) *Brownian motion on the line*, PhD dissertation, Univ. Wiskonsin, Madison.
  - Y. Le Jan, O. Raimond (2004) *Flows, coalescence and noise*, The Annals of Probability 32, no. 2, 1247—1315.
  - A.A. Dorogovtsev (2004) *One Brownian stochastic flow*, Theory of Stochastic Processes, **10(26)**, no. 3–4, 21–25.

Some history of the system of diffusion particles with interaction

- ⋆ The cases of finite and infinite numbers of particles with sticking that have mass and speed and their motion obey the laws of mass conservation and inertion
  - E. Weinan, Yu.G. Rykov, Ya.G. Sinai (1996) *Generalized Variational Principles, Global Weak Solutions and Behavior with Random Initial Data for Systems of Conservation Laws Arising in Adhesion Particle Dynamics*, Communications in Mathematical Physics **177**, 349–380.
- ⋆ empiric distribution the set of N processes with interection for fixed time
  - V. Malyshev, A.D. Manita (2006) *Asymptotic Behaviour in the Time Synchronization Model*, In Representation Theory, Dynamical Systems, and Asymptotic Combinatorics AMS, American Mathematical Society Translations - Series 2 Advances in the Mathematical Sciences, **217**, 101–115.

Non-random initial mass distribution

Theorem 1.

There exists system of processes $X = \{x(k,t);\ k \in \mathbb{Z},\ t \geq 0\}$ *such that*

1) $x(k,\cdot)$ *is continuous square integrable local martingale with respect to*

$$(\mathcal{F}_t)_{t\geq 0} = (\sigma(x(k,s);\ s \leq t,\ k \in \mathbb{Z}))_{t\geq 0};$$

2) $x(k,0) = x_k, \quad k \in \mathbb{Z};$

3) $\forall\ k \in \mathbb{Z}\ \ \forall\ t \geq 0 \quad x(k,t) \leq x(k+1,t);$

4) $\forall\ t \geq 0 \quad \langle x(k,\cdot)\rangle_t = \int\limits_0^t \frac{ds}{m(k,s)},$

where

$$m(k,t) = \sum_{i \in |A(k,t)|} a_i,$$

$$A(k,t) = \{j \in \mathbb{Z}:\ \exists\ s \leq t,\ x(k,s) = x(j,s)\};$$

5) *the joint characteristic*

$$\langle x(l,\cdot), x(k,\cdot)\rangle_t\, \mathbb{I}_{\{t<\tau_{l,k}\}} = 0,$$

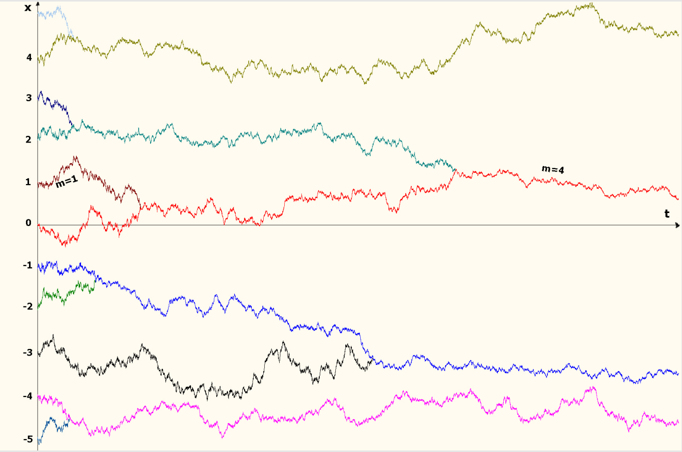
where

$$\tau_{l,k} = \inf\{t:\ x(l,t) = x(k,t)\}.$$

Idea of construction

1. For finite family.
2. Stabilization.

Finite family



Idea of construction, stabilization

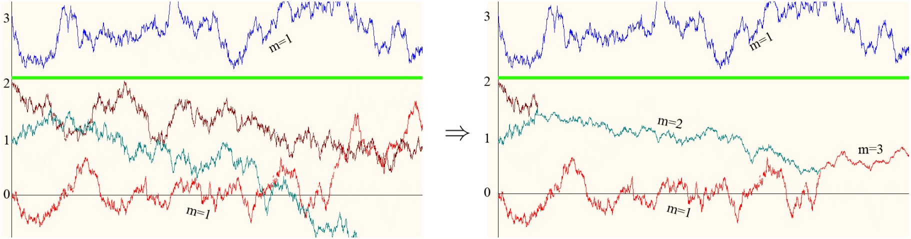
Lemma.

Let $\{w_k;\ k \in \mathbf{N} \cup \{0\}\}$ *be the system of standard independent Wiener processes. Denote*

$$\xi_k = k + \max_{t\in[0,1]} w_k(t), \quad \eta_k = k + \min_{t\in[0,1]} w_k(t).$$

Then for every $\delta \in (0, \frac{1}{2})$

$$\mathbf{P}\left\{\overline{\lim_{n\to\infty}}\left\{\max_{k=\overline{0,n}} \xi_k \le n + \tfrac{1}{2},\ \eta_{n+1} > n + \tfrac{1}{2} + \delta\right\}\right\} = 1.$$



Theorem 2.

$X = (\dots, x(-n, \cdot), \dots, x(n, \cdot), \dots)$ and $Y = (\dots, y(-n, \cdot), \dots, y(n, \cdot), \dots)$ satisfy conditions 1)–5). Then $X \overset{d}{=} Y$.

Properties

1°. Sticking:

$$(x(k,t) - x(l,t))\mathbb{I}_{\{t > \tau_{l,k}\}} = 0.$$

2°. Mass growth:

$$\mathbf{P}\left\{ \overline{\lim_{t \to +\infty}} \frac{m(k,t)}{4\sqrt{t \operatorname{lln} t}} \le 1 \right\} = 1.$$

3°. Finite time of any two particles to sticking:

$$\mathbf{P}\{\tau_{k,k+p} < +\infty\} = 1.$$

Theorem 3.

Let $\mu = \sum\limits_{k\in\mathbb{Z}} a_k \delta_{x_k}$ *be a stationary point measure on* $\mathbb{R}$ *such that* $\sum\limits_{k\in\mathbb{Z}} \mathbb{I}_{[a,b]}(x_k) < \infty$ *for every* $a, b \in \mathbb{R}$. *Then there exists a system of processes* $\{x(k,t);\ k\in\mathbb{Z},\ t\geq 0\}$ *such that*

1) $x(k,\cdot) - x_k$ *is continuous square integrable local martingale respect to*

$$(\mathcal{F}_t)_{t\geq 0} = (\sigma(x(k,s);\ s\leq t,\ k\in\mathbb{Z}))_{t\geq 0};$$

2) $x(k,0) = x_k, \quad k\in\mathbb{Z};$
3) $\forall\ k\in\mathbb{Z}\ \ \forall\ t\geq 0 \quad x(k,t)\leq x(k+1,t);$
4) $\forall\ t\geq 0 \quad \langle x(k,\cdot) - x_k\rangle_t = \int\limits_0^t \frac{ds}{m(k,s)},$

   where

$$m(k,t) = \sum_{i\in|A(k,t)|} a_i,$$

$$A(k,t) = \{j\in\mathbb{Z}:\ \exists\ s\leq t,\ x(k,s) = x(j,s)\};$$

5) *the joint characteristic*

$$\langle x(l,\cdot) - x_l, x(k,\cdot) - x_k\rangle_t\, \mathbb{I}_{\{t<\tau_{l,k}\}} = 0,$$

   where

$$\tau_{l,k} = \inf\{t:\ x(l,t) = x(k,t)\}.$$

$$\mu_t = \sum_{k \in \mathbb{Z}} a_k \delta_{x(k,t)}$$

Remake.

If μ_0 is stationary measure with respect to spatial shifts. Then μ_t is stationary measure for each $t > 0$ too.

Markov property of μ_t

Theorem 4.

$\{\mu_t;\ t \geq 0\}$ is continuous Markov process in the space of general functions $\mathcal{D}'(\mathbb{R})$. Moreover, for every $\varphi \in C_0^2(\mathbb{R})$ and $f \in C^2(\mathbb{R})$

$$F(\mu_t) - F(\mu_0) - \frac{1}{2}\int_0^t AF(\mu_s)ds = \mathfrak{M}_F(t),$$

where

$$F(\mu_t) = f(\langle \mu_t, \varphi \rangle),$$
$$AF(\mu_t) = f''(\langle \mu_t, \varphi \rangle)\langle \mu_t, \dot{\varphi}^2 \rangle + f'(\langle \mu_t, \varphi \rangle)\langle \mu_t^*, \varphi'' \rangle$$
$$\mu_t^* = \sum_{k \in \mathbb{Z}} \delta_{x(k,t)}$$

and $\mathfrak{M}_F$ is continuous square integrable local martingale.