Exercise list 3

Gromov's Lemma

1. Let X be a simply connected $C'(\frac{1}{6})$ complex. Show that its 2-cells are embedded.
2. Let X be a simply connected $C'(\frac{1}{6})$ complex and let $D \to X$ be a reduced disc diagram. Show that ∂D has at least as many edges as the biggest 2-cell in D.
3. Let X be a simply connected $C'(\frac{1}{6})$ complex. Show that its 2-cells are isometrically embedded.

Helly property

4. Let X be a simply connected $C'(\frac{1}{6})$ complex. Let R_1, R_2 and R_3 be pairwise intersecting 2-cells of X. Show that $R_1 \cap R_2 \cap R_3$ is non-empty and connected.
5. Let X be a simply connected $C'(\frac{1}{6})$ complex and let $\{R_i\}_{i=0,\dots,n}$ be a finite family of pairwise intersecting 2-cells of X.
   (a) Show that $R_0 \cap (R_1 \cup \dots \cup R_n)$ is non-empty and connected.
   (b) Conclude that $\cap_i R_i \neq \emptyset$.