

MUSTERLÖSUNG: AUFGABE 28

Es soll gelten:

$$y_1c_1 + y_2c_2 + y_3c_3 = x_1b_1 + x_2b_2 + x_3b_3.$$

$$\Leftrightarrow y_1 \cdot \begin{pmatrix} c_{11} \\ c_{12} \\ c_{13} \end{pmatrix} + y_2 \cdot \begin{pmatrix} c_{21} \\ c_{22} \\ c_{23} \end{pmatrix} + y_3 \cdot \begin{pmatrix} c_{31} \\ c_{32} \\ c_{33} \end{pmatrix} = x_1 \cdot \begin{pmatrix} b_{11} \\ b_{12} \\ b_{13} \end{pmatrix} + x_2 \cdot \begin{pmatrix} b_{21} \\ b_{22} \\ b_{23} \end{pmatrix} + x_3 \cdot \begin{pmatrix} b_{31} \\ b_{32} \\ b_{33} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} y_1 \cdot c_{11} + y_2 \cdot c_{21} + y_3 \cdot c_{31} \\ y_1 \cdot c_{12} + y_2 \cdot c_{22} + y_3 \cdot c_{32} \\ y_1 \cdot c_{13} + y_2 \cdot c_{23} + y_3 \cdot c_{33} \end{pmatrix} = \begin{pmatrix} x_1 \cdot b_{11} + x_2 \cdot b_{21} + x_3 \cdot b_{31} \\ x_1 \cdot b_{12} + x_2 \cdot b_{22} + x_3 \cdot b_{32} \\ x_1 \cdot b_{13} + x_2 \cdot b_{23} + x_3 \cdot b_{33} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} c_{11} \cdot y_1 + c_{21} \cdot y_2 + c_{31} \cdot y_3 \\ c_{12} \cdot y_1 + c_{22} \cdot y_2 + c_{32} \cdot y_3 \\ c_{13} \cdot y_1 + c_{23} \cdot y_2 + c_{33} \cdot y_3 \end{pmatrix} = \begin{pmatrix} b_{11} \cdot x_1 + b_{21} \cdot x_2 + b_{31} \cdot x_3 \\ b_{12} \cdot x_1 + b_{22} \cdot x_2 + b_{32} \cdot x_3 \\ b_{13} \cdot x_1 + b_{23} \cdot x_2 + b_{33} \cdot x_3 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{pmatrix} \cdot \vec{y} = \begin{pmatrix} b_{11} & b_{21} & b_{31} \\ b_{12} & b_{22} & b_{32} \\ b_{13} & b_{23} & b_{33} \end{pmatrix} \cdot \vec{x}$$

$$\Leftrightarrow \vec{y} = (C^{-1} \circ B)\vec{x} \text{ mit } C = \begin{pmatrix} 1 & -1 & -2 \\ 1 & 3 & 7 \\ 2 & 3 & 6 \end{pmatrix} \text{ und } B = \begin{pmatrix} 1 & 2 & 2 \\ -1 & 3 & 3 \\ 2 & 7 & 6 \end{pmatrix}$$

Berechne  $C^{-1}$ .

$$\left( \begin{array}{ccc|ccc} 1 & -1 & -2 & 1 & 0 & 0 \\ 1 & 3 & 7 & 0 & 1 & 0 \\ 2 & 3 & 6 & 0 & 0 & 1 \end{array} \right)$$

II=II-I

$$\left( \begin{array}{ccc|ccc} 1 & -1 & -2 & 1 & 0 & 0 \\ 0 & 4 & 9 & -1 & 1 & 0 \\ 2 & 3 & 6 & 0 & 0 & 1 \end{array} \right)$$

III=III-2·I

$$\left( \begin{array}{ccc|ccc} 1 & -1 & -2 & 1 & 0 & 0 \\ 0 & 4 & 9 & -1 & 1 & 0 \\ 0 & 5 & 10 & -2 & 0 & 1 \end{array} \right)$$

III=4·III-5·II

$$\left( \begin{array}{ccc|ccc} 1 & -1 & -2 & 1 & 0 & 0 \\ 0 & 4 & 9 & -1 & 1 & 0 \\ 0 & 0 & -5 & -3 & 5 & 4 \end{array} \right)$$

II=II·1/4; III=III·1/5

$$\left( \begin{array}{ccc|ccc} 1 & -1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 9/4 & -1/4 & 1/4 & 0 \\ 0 & 0 & 1 & 3/5 & 1 & -4/5 \end{array} \right)$$

$$\text{II} = \text{II} - 9/4 \cdot \text{III}$$

$$\left( \begin{array}{ccc|ccc} 1 & -1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -8/5 & -2 & 9/5 \\ 0 & 0 & 1 & 3/5 & 1 & -4/5 \end{array} \right)$$

$$\text{I} = \text{I} + \text{II} + 2 \cdot \text{III}$$

$$\left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 3/5 & 0 & 1/5 \\ 0 & 1 & 0 & -8/5 & -2 & 9/5 \\ 0 & 0 & 1 & 3/5 & 1 & -4/5 \end{array} \right)$$

$$\Rightarrow C^{-1} = \begin{pmatrix} 3/5 & 0 & 1/5 \\ -8/5 & -2 & 9/5 \\ 3/5 & 1 & -4/5 \end{pmatrix}$$

Berechne  $\vec{y}$ .

$$\begin{aligned} \vec{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} &= (C^{-1}B) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \\ &= \left( \left( \begin{pmatrix} 3/5 & 0 & 1/5 \\ -8/5 & -2 & 9/5 \\ 3/5 & 1 & -4/5 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 2 \\ -1 & 3 & 3 \\ 2 & 7 & 6 \end{pmatrix} \right) \right) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 13/5 & 12/5 \\ 4 & 17/5 & 8/5 \\ -2 & -7/5 & -3/5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \\ &= \begin{pmatrix} x_1 + 13/5x_2 + 12/5x_3 \\ 4x_1 + 17/5x_2 + 8/5x_3 \\ -2x_1 + -7/5x_2 + -3/5x_3 \end{pmatrix} \end{aligned}$$

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