Geometry Day 2023 in Leipzig

Abstracts [1]

- **Evgenii Antonov** (Jena)

  Applications of Nijenhuis geometry in the theory of F-manifolds

  F-manifolds, introduced by Hertling and Manin as a generalization of Frobenius manifolds, are an important class of manifolds that have gained attention in recent years. They are equipped with a commutative, associative multiplication operation $\circ$ with unit element e defined on the tangent bundle, which satisfies a natural integrability condition. The presence of an Euler vector field E, corresponding to the F-structure, induces the operator field $L = E\circ$ with vanishing Nijenhuis torsion. The study of F-manifolds has seen exciting developments and has become an active area of research. Nijenhuis geometry has emerged as a promising tool for investigating the geometry of these manifolds. The aim of this talk is to establish a connection between the theory of F-manifolds and Nijenhuis geometry, and to show how the application of Nijenhuis geometry has led to exciting new developments in the study of F-manifolds, including the discovery of new examples

- **Andreas Bernig** (Frankfurt a.M.)

  The Weyl principle in pseudo-Riemannian geometry

  The classical Weyl principle states that the volume of a tube around a compact submanifold of euclidean space is a polynomial in the radius. The (suitably normalized) coefficients depend only on the intrinsic geometry of the manifold and not on the embedding. They are called intrinsic volumes and comprise as special cases the volume, the total scalar curvature, and the Euler characteristic. Using Alesker's theory of valuations on manifolds we construct intrinsic volumes of pseudo-Riemannian manifolds that satisfy a version of Weyl's principle. The intrinsic volumes can be extended to certain manifolds with a sign-changing metric and satisfy a version of the Chern-Gauss-Bonnet theorem. This is joint work with D. Faifman and G. Solanes.

- **Katherina von Dichter** (TU München)

  The Diameter-width-ratio for complete and pseudo-complete sets

  Complete sets in general Minkowski spaces are convex sets with the property that adding any additional point to the set enlarges its diameter. This property holds for all constant width bodies, and thus constant width always implies completeness, while the opposite is in general not true. Pseudo-complete sets are convex sets, for which the diameter is equal to the sum of the in- and circumradius. Moreover, pseudo-completeness implies completeness. We present results on the diameter-width-ratio for complete and pseudo-complete sets, depending on the Minkowski asymmetry measures of the convex body, which itself is bounded by the dimension of the ambient space.

- **Samantha Fairchild** (MPI Leipzig)

  Average degree of the essential variety

  The essential variety is an algebraic subvariety of dimension 5 in real projective space which encodes the relative pose of two calibrated pinhole cameras. The 5-point algorithm in computer vision computes the real points in the intersection of the essential variety with a linear space of codimension 5. The degree of the essential variety is 10, so this intersection consists of 10 complex points in general. We will present the expected number of real intersection points with respect to different probability distributions, and some interesting geometry and computational work that arises from this problem. This is based on joint work with Paul Breiding, Pierpaola Santarsiero, and Elima Shehu.

- **Dirk Frettlöh** (Bielefeld)

  Tilings with transcendental inflation factor

  Tile substitutions are a fundamental tool to construct aperiodic tilings like the Penrose tilings. Such a tile substitution is a rule how to inflate the prototiles (the building blocks of the tiling) by some common factor and dissect the inflated tiles into copies of the original prototiles. If the number of prototiles is finite the inflation factor is necessarily an algebraic number. Here we present the first examples of tile substitutions where the inflation factor is a transcendental number.

[1] Version 2023-03-07, `www.math.uni-leipzig.de/geometrietag`

- **Nadine Große** (Freiburg)

  On the L^p-spectrum of the Dirac operator

  We study the p-dependence of the spectrum of the Dirac operator. In particular, we give sufficient conditions for the L^p-spectrum to be independent on p on noncompact manifolds. As an application we use this result to compute the L^2-spectrum of classes of manifolds by calculating the L^1-spectrum. This is joint work with Nelia Charalambous.

- **Mario Kummer** (Dresden)

  Convex algebraic geometry and Riemannian geometry

  We explain how techniques from the emerging field of convex algebraic geometry can be used to obtain new restrictions on the topology of Riemannian manifolds satisfying certain sectional curvature bounds. Conversely, we show how the set of algebraic curvature operators satisfying a sectional curvature bound give new counterexamples to a recently refuted conjecture in convex algebraic geometry.

- **Philippe Kupper** (KIT)

  String topology of symmetric spaces

  On the homology of the free loop space of a closed manifold M there exists the so-called Chas-Sullivan product. It is a product defined via the concatenation of loops and can, for example, be used to study closed geodesics of Riemannian or Finsler metrics on M. In this talk I will outline how one can use the geometry of symmetric spaces to partially compute the Chas-Sullican product. In particular, we will see that the powers of certain non-nilpotent classes correspond to the iteration of closed geodesics in a symmetric metric. If time permits, some results on the Gorseky-Hingston cohomology product will also be mentioned. This talk is based on joint work with Maximilian Stegemeyer (`https://arxiv.org/abs/2212.09350`).

- **Léo Mathis** (Frankfurt a.M.) ***The zonoid algebra and its application***

  In a joint work with P. Breiding P. Bürgisser and A. Lerario, we define a multiplicative structure on the space of zonoid (a particular class of convex bodies) that turns it into an algebra. I will show how this zonoid algebra is linked to the theory of random matrices and allows to produce new inequalities on expected absolute random determinants. Moreover, I will explain how this algebra can compute independent random intersection in Riemannian homogeneous spaces and thus can be seen as a *probabilistic cohomology ring.* I will show how this produces for example a formula for the self intersection of submanifold of real codimension 2 in complex projective space that generalizes Bézout's formula for complex hypersurfaces.

- **Marius Müller** (Freiburg/Leipzig)

  The Willmore flow of tori of revolution

  The Willmore energy measures the bending of surfaces. Its L2 gradient flow is an object of interest in geometric analysis. While some convergence results exist for the flow of topological spheres, little is known about topological tori.

  We provide some insights by looking at the evolution of tori of revolution. In this context there appears surprisingly (or not surprisingly (?)) the hyperbolic geometry.

  I will try to convince you why this is natural and discuss the convergence results we obtained.

  This is a joint work with A. Dall'Acqua (Ulm), R. Schätzle (Tübingen) and A. Spener (Ulm)

- **Sayan Mukherjee** (MPI Leipzig/ScaDS.AI)

  A Sheaf Theoretic Framework for Modeling Shapes

  We will consider modeling shapes and fields via topological and lifted-topological transforms. Specifically, we show how the Euler Characteristic Transform and the Lifted Euler Characteristic Transform can be used in practice for statistical analysis of shape and field data. We also state a moduli space of shapes for which we can provide a complexity metric for the shapes. We also provide a sheaf theoretic construction of shape space that does not require diffeomorphisms or correspondence. A direct result of this sheaf theoretic construction is that in three dimensions for meshes, 0-dimensional homology is enough to characterize the shape. Time permitting we will discuss stochastic processes on fibers and an initial attempt to define stochastic processes on sheaves.

- **Jesse Ratzkin** (Würzburg)

  Constant Q-curvature metrics with isolated singularities

  The Q-curvarture of a Riemannian metric is a scalar one constructs using its scalar and Ricci curvatures which transforms nicely after a conformal transformation. The PDE prescribing the Q-curvature of a conformal metric is a higher-order analog of the well-known corresponding equation for scalar curvature, and some much recent work concentrates on reproducing results for scalar curvature in the Q-curvature setting. I will present some of these results for metrics with isolated singularities, particularly a general construction of new solutions and a compactness theorem for certain subsets of the corresponding moduli space.

- **Darya Sukhorebska** (Kharkiv/Münster)

Simple closed geodesics on regular tetrahedra in spaces of constant curvature

(joint work with Alexander A. Borisenko)

In Euclidean three-dimensional space the faces of a tetrahedron have zero Gaussian curvature, and the curvature of a tetrahedron is concentrated only on its vertices. A complete classification of closed geodesics on a regular tetrahedron in Euclidean space follows from a tiling of Euclidean plane with regular triangles.

We studied simple closed geodesics on regular tetrahedra in three dimensional hyperbolic and spherical spaces. In hyperbolic or spherical space the Gaussian curvature of faces is $k = -1$ or 1 respectively. Therefore, the curvature of a tetrahedron is determined not only by vertices, but also by faces. In hyperbolic space the planar angle α of the faces of a regular tetrahedron satisfies $0 < \alpha < \pi/3$. In spherical space the planar angle α satisfies $\pi/3 < \alpha < 2\pi/3$. In both cases the intrinsic geometry of a tetrahedron depends on the planar angle. The behaviour of closed geodesics on a regular tetrahedron in three dimensional spaces of constant curvature k depending on the sign of k.

A simple closed geodesic γ on a tetrahedron is of type (p, q) if γ has p points on each of two opposite edges of the tetrahedron, q points on each of two other opposite edges, and $(p + q)$ points on each of the remaining two opposite edges. We proved that on a regular tetrahedron in hyperbolic space for any coprime integers $(p, q), 0 \leq p < q$, there exists unique, up to the rigid motion of the tetrahedron, simple closed geodesic of type (p, q). This geodesic passes through the midpoints of two pairs of opposite edges of the tetrahedron. Geodesics of type (p, q) exhaust all simple closed geodesics on a regular tetrahedron in hyperbolic space. The number of simple closed geodesics of length bounded by L has order of grows $c(\alpha)L^2$, when L tends to infinity (see [1]).

If the planar angles of any tetrahedron in hyperbolic space are at most $\pi/4$, then for any pair of coprime integers (p, q) there exists a simple closed geodesic of type (p, q) (see [3]). This situation is different from the Euclidean space, where in general there is no simple closed geodesic on a non-regular tetrahedron. On a regular tetrahedron in spherical space there exists a finite number of simple closed geodesics. The length of all these geodesics is less than 2π. For any coprime integers (p, q) we found the numbers α_1 and α_2, satisfying the inequalities $\pi/3 < \alpha_1 < \alpha_2 < 2\pi/3$, such that

1. if $\pi/3 < \alpha < \alpha_1$, then on a regular tetrahedron in spherical space with the planar angle α there exists the unique simple closed geodesic of type (p, q), up to the rigid motion of this tetrahedron. This geodesic passes through the midpoints of two pairs of opposite edges of the tetrahedron;
2. if $\alpha_2 < \alpha < 2\pi/3$, then on a regular tetrahedron with the planar angle α there is 1 not simple closed geodesic of type (p, q)(see [2]).

In [3], the necessary and sufficient condition for the existence of a simple closed geodesic on a regular tetrahedron in S3 is presented.

References

[1] A. A. Borisenko, D. D. Sukhorebska, Simple closed geodesics on regular tetra- hedra in Lobachevsky space, Sb. Math., 211:5 (2020), 617-642. `http://dx.doi.org/10.1070/SM9212`

[2] A. A. Borisenko, D. D. Sukhorebska, Simple closed geodesics on regular tetra- hedra in spherical space, Sb. Math, 212:8 (2021), 1040-1067. `https://doi.org/10.1070/SM9433`

[3] A. A. Borisenko, A necessary and sufficient condition for the existence of simple closed geodesics on regular tetrahedra in spherical space, Sb. Math., 213:2 (2022), 161-172

`https://doi.org/10.1070/SM95762`

- **Thomas Wannerer** (Jena)

Valuations in affine convex geometry

In convex geometry, the maps that assign to a convex body its difference body, projection body, or volume have the following properties: (1) invariance under volume-preserving linear changes of coordinates; (2) continuity; and (3) finite additivity. We explore the question whether there exist other constructions with these properties. We discover a surprising dichotomy: There are no new examples if one assumes also invariance under translations, but there is a plethora of examples without this assumption.

Based on joint work with Jakob Henkel.