Summary of "On matrix generalization of Hurwitz polynomials" by Xuzhou Zhan

This thesis focuses on matrix generalizations of Hurwitz polynomials. A real polynomial with all its roots in the open left half plane of the complex plane is called a Hurwitz polynomial. The study of these Hurwitz polynomials has a long and abundant history, which is associated with the names of Hermite, Routh, Hurwitz, Liénard, Chipart, Wall, Gantmacher et al.

The direct matricial generalization of Hurwitz polynomials is naturally defined as follows: A $p \times p$ matrix polynomial F is called a Hurwitz matrix polynomial if $\det F$ is a Hurwitz polynomial. Recently, Choque Rivero followed another line of matricial extensions of the classical Hurwitz polynomial, called matrix Hurwitz type polynomials. However, the notion "matrix Hurwitz type polynomial" is still irrelative to "Hurwitz matrix polynomial" due to the totally unclear zero location of the former notion. So the main goal of this thesis is to discover the relation between the two notions "matrix Hurwitz-type polynomials" and "Hurwitz matrix polynomials" and provide some criteria to identify Hurwitz matrix polynomials.

The central idea is to determine the inertia triple of matrix polynomials in terms of some related matrix sequences. Suppose that F is a $p \times p$ matrix-valued polynomial of degree n. We split F into the odd part $F_{\langle o \rangle}$ and the even part $F_{\langle e \rangle}$, which allow us to introduce an essential rational matrix functions of right type G. From the matrix coefficients of the Laurent series of G we construct the (n-1)-th extended sequence of right Markov parameters (SRMP) of F. Then we show that the inertia triple of F can be characterized by a combination of the inertia triples of two block Hankel matrices $\mathbf{H}^{\langle 0 \rangle}_{[\frac{n-1}{2}]}$ and $\mathbf{H}^{\langle 1 \rangle}_{[\frac{n}{2}]-1}$ and the number of zeros (counting for multiplicities) of greatest right common divisors of $F_{\langle e \rangle}$ and $F_{\langle o \rangle}$ lying on the interval $(-\infty, 0]$. By an analogous approach we also obtain the dual results for the inertia triple of F in terms of the SLMP of F. Then we demonstrate that F is a Hurwitz matrix polynomial of degree n if and only if the $(n-1)$-th SRMP (resp. SLMP) of F is a Stieltjes positive definite sequence. On this account, the two notions "Hurwitz matrix polynomials" and "matrix Hurwitz type polynomials" are equivalent.

In addition, we investigate quasi-stable matrix polynomials appearing in the theory of stability, which contain Hurwitz matrix polynomials as a special case. We seek a correspondence between quasi-stable matrix polynomials, Stieltjes moment problems and multiple Nevanlinna-Pick interpolation in the Stieltjes class. Accordingly, we prove that F is a quasi-stable matrix polynomial if and only if the $(n-1)$-th SRMP (resp. SLMP) of F is a Stieltjes non-negative definite extendable sequence and the zeros of right (resp. left) greatest common divisors of $F_{\langle e \rangle}$ and $F_{\langle o \rangle}$ are located on the interval $(-\infty, 0]$.