Abstract

We first investigate the uncertainty relations based on the sum of variances and derive a family of weighted uncertainty relations to provide an optimal lower bound for all situations. Our work indicates that it seems unreasonable to assume a priori that incompatible observables have equal contribution to the variance-based sum form uncertainty relations. We also study the role of mutually exclusive physical states in the recent work and generalize the variance-based uncertainty relations to mutually exclusive uncertainty relations.

Next, we develop a new kind of entanglement detection criteria within the framework of marjorization theory and its matrix representation. By virtue of majorization uncertainty bounds, we are able to construct the entanglement criteria which have advantage over the scalar detecting algorithms as they are often stronger and tighter.

Furthermore, we explore various expression of entropic uncertainty relations, including sum of Shannon entropies, majorization uncertainty relations and uncertainty relations in presence of quantum memory. For entropic uncertainty relations without quantum side information, we provide several tighter bounds for multi-measurements, with some of them also valid for Rényi and Tsallis entropies besides the Shannon entropy. We employ majorization theory and actions of the symmetric group to obtain an admixture bound for entropic uncertainty relations with multi-measurements. Comparisons among existing bounds for multi-measurements are also given. However, classical entropic uncertainty relations assume there has only classical side information. For modern uncertainty relations, those who allowed for non-trivial amount of quantum side information, their bounds have been strengthened by our recent result for both two and multi-measurements.

Finally, we propose an approach which can extend all uncertainty relations on Shannon entropies to allow for quantum side information and discuss the applications of our entropic framework. Combined with our uniform entanglement frames, it is possible to detect entanglement via entropic uncertainty relations even if there is no quantum side information. With the rising of quantum information theory, uncertainty relations have been established as important tools for a wide range of applications, such as quantum cryptography, quantum key distribution, entanglement detection, quantum metrology, quantum speed limit and so on. It is thus necessary to focus on the study of uncertainty relations.