Analysis of a Two-dimensional Nonlinear Sigma model with Gravitino
Abstract

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In this dissertation we considered a nonlinear sigma model with gravitino field. This is a supersymmetric extension of the nonlinear sigma model in the string theory, and we set up the geometric model using commuting variables, such that we could analyze it using the tools from calculus of variations.

We introduced an action functional which corresponds to the super harmonic map functional, which has four arguments: a map between Riemannian manifolds, a vector spinor, a Riemannian metric and a gravitino. After getting the total variation formula, we considered the symmetries that the action functional possesses. By Noether's principle these families of symmetries induces conservation laws, which help to interpret the energy-momentum tensor and the supercurrent as holomorphic sections of some complex bundle. We also discussed the supersymmetry of our model. It turns out that the supersymmetry only remains in some particular cases, which is still useful in the analysis.

Then we defined the weak solution in the distributional sense, and using Riesz potential estimates and Riviere regularity theory, we could improve the regularity of the weak solutions. More precisely, when the Riemannian metric and the gravitino are smooth, then any weak solution is actually smooth; and when the gravitino are coarse but subcritical, we can still show that the weak solutions are Holder continuous.

Next we considered the compactness of solutions with bounded energies. We showed the small energy regularity on local domains and gap properties on the global surface. We also established the Pohozaev identities and thus showed the removable singularity theorem. Finally, for a sequence of solutions of uniformly bounded energies with respect to a converging sequence of gravitino fields, we showed that they converges weakly. Actually away from finite points, the convergence is strong and at those points, the energies concentrate. After a rescaling, each of these points corresponds to finitely some Dirac-harmonic maps with curvature terms defined on the Riemann sphere. Moreover, we established the energy identities along the weakly convergent sequences modulo these bubbles.