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Branching Processes: Optimization, Variational Characterization, and Continuous Approximation

Abstract

In this thesis, we use multitype Galton-Watson branching processes in random environments as individual-based models for the evolution of structured populations with both demographic stochasticity and environmental stochasticity, and investigate the phenotype allocation problem. We explore a variational characterization for the stochastic evolution of a structured population modeled by a multitype Galton-Watson branching process. When the population under consideration is large and the time scale is fast, we deduce the continuous approximation for multitype Markov branching processes in random environments.

Many problems in evolutionary biology involve the allocation of some limited resource among several investments. It is often of interest to know whether, and how, allocation strategies can be optimized for the evolution of a structured population with randomness. In our work, the investments represent different types of offspring, or alternative strategies for allocations to offspring. As payoffs we consider the long-term growth rate, the expected number of descendants with some future discount factor, the extinction probability of the lineage, or the expected survival time. Two different kinds of population randomness are considered: demographic stochasticity and environmental stochasticity. In chapter 2, we solve the allocation problem w.r.t. the above payoff functions in three stochastic population models depending on different kinds of population randomness.

Evolution is often understood as an optimization problem, and there is a long tradition to look at evolutionary models from a variational perspective. In chapter 3, we deduce a variational characterization for the stochastic evolution of a structured population modeled by a multitype Galton-Watson branching process. In particular, the so-called retrospective process plays an important role in the description of the equilibrium state used in the variational characterization. We define the retrospective process associated with a multitype Galton-Watson branching process and identify it with the mutation process describing the type evolution along typical lineages of the multitype Galton-Watson branching process.

Continuous approximation of branching processes is of both practical and theoretical interest. However, to our knowledge, there is no literature on approximation of multitype branching processes in random environments. In chapter 4, we firstly construct a multitype Markov branching process in a random environment. When conditioned on the random environment, we deduce the Kolmogorov equations and the mean matrix for the conditioned branching process. Then we introduce a parallel mutation-selection Markov branching process in a random environment and analyze its instability property. Finally, we deduce a weak convergence result for a sequence of the parallel Markov branching processes in random environments and give examples for applications.