Asymptotic behaviour of capillary problems governed by disjoining pressure potentials

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Abstract

The behaviour of capillary effects in porous materials is often considered in physical chemistry. Porous matter contains a filigree network of pores which are intricate and affiliated into each other. If these porous materials get in contact with a liquid, capillary effects lead to an adsorption of the liquid. A major task is to determine the absorbed amount of substance in the pores. Therefore it is necessary to have statistics about the distribution of the shape and size of the occurring cavities. Each category of pores can absorb a fixed condensate quantity. To get the desired result for a specific type of pore, one examines in each case a single pore. Thus combining the statistics of the cavities with the specific condensate quantity yields the total quantity of the absorbed liquid.

If the total quantity of absorbed liquid as a function of gas pressure is known one can come to conclusions about the distribution of the pores and their sizes. Knowing these distributions enables us to understand the structure of porous materials. This structure affects a lot of interesting material properties, for example elasticity, stability, absorbing capacity of liquids and permeability of sieves. So research in this field will lead to a lot of applications in physical chemistry.

Some kind of pores can be modeled as cylinders over some domain Ω. In my dissertation I consider these *vapour//liquid//solid* configurations where the Hamaker constant becomes *negative*. The configuration *vapour nitrogen//liquid nitrogen//quartz* is a generic example. But instead of nitrogen, other nonpolar molecules like oxygen, hydrogen or rare gases are also possible. In this case the disjoining pressure potential becomes an additional term in the classic capillary equation, which means that we get the following boundary value problem

$$\begin{aligned} \operatorname{div} \mathbf{T}\, u &= \kappa u + F(x,u) + \lambda && \text{in } \Omega, \\ \nu \cdot \mathbf{T}\, u &= 1 && \text{on } \Sigma. \end{aligned} \tag{1}$$

Thereby we have

$$F = c \int\limits_{\Omega_s} \left[(x_1 - y_1)^2 + (x_2 - y_2)^2 + (u - y_3)^2 \right]^{-p/2} \mathrm{d}y, \qquad p > 3 \tag{2}$$

the *disjoining pressure potential*, c is a negative constant and Ω_s denotes the solid domain.

The aim of the work is to determine the qualitative ascent of the fluid on the walls of the pores. The principal point is the extension of a comparison principle of Concus and Finn to the current circumstances, which is:

Theorem 1 *Let $\kappa > 0$ and suppose $\Sigma = \partial\Omega$ admits a decomposition $\Sigma = \Sigma_\alpha \cup \Sigma_\beta \cup \Sigma_0$, where $\Sigma_\beta \in C^1$ and Σ_0 has one-dimensional Hausdorff measure zero. Let $u, v \in C^2(\Omega) \cap C^1(\Sigma_\beta \cup \Omega)$ with the properties*

(i)	$\operatorname{div} \boldsymbol{T}u - \kappa u - F(x,u)$	$\geq$	$\operatorname{div} \boldsymbol{T}v - \kappa v - F(x,v)$	*in* Ω,
(ii)	u	$\leq$	v	*as* $x \to \Sigma_\alpha$,
(iii)	$\nu \cdot \boldsymbol{T}u$	$\leq$	$\nu \cdot \boldsymbol{T}v$	*as* $x \to \Sigma_\beta$.

Then we have $u \leq v$ in Ω.

So for straight cylindrical cavities with arbitrary cross-section, a comparison principle is obtained. For some special cases (circular cross-section, horizontal wall, parallel plates, wedge) explicit results for the asymptotic behaviour are stated. In detail, if d denotes the distance to the boundary of Ω and v the solution of (1), we have the following asymptotic result

$$v(d) \sim C \cdot d^{3-p}, \qquad \text{as } d \to 0, \tag{3}$$

for some positive constant C.

A second result, which easily follows from the above-mentioned comparison principle, is that the solution of (1) is unique.