Impulse Control for Jump-Diffusions: Viscosity Solutions of Quasi-Variational Inequalities and Applications in Bank Risk Management

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Summary

This work is about combined stochastic and impulse control for jump-diffusions, its relation to Hamilton-Jacobi-Bellman quasi-variational inequalities (HJBQVIs), their numerical solution, and applications in bank risk management. An HJBQVI is a nonlinear partial integro-differential equation (PIDE) consisting of an HJB part (for stochastic control) combined with a nonlocal impulse intervention term; the jump measure in the PIDE may be singular, corresponding to a jump process of infinite activity.

General theorems for existence and uniqueness of viscosity solutions of such HJBQVIs are established in Chapter 2. We prove via stochastic means that the value function of stochastic and impulse control is an HJBQVI viscosity solution, whereas our uniqueness (comparison) results adapt techniques from viscosity solution theory. To our knowledge, our results are the first rigorous treatment of impulse control for jump-diffusion processes in a general viscosity solution framework. In the proofs, no prior continuity of the value function is assumed, quadratic costs are allowed, and elliptic and parabolic results are presented for solutions possibly unbounded at infinity.

In Chapter 3, we analyze the numerical solution of an HJBQVI via iterated optimal stopping. First, the viscosity characteristics of the value function of combined stochastic control and stopping is established by similar techniques as in Chapter 2. We prove that the value functions of iterated optimal stopping as solutions of suitable HJB variational inequalities converge to the unique HJBQVI viscosity solution.

As an application of stochastic and impulse control, in Chapter 4 we propose a dynamic model where a bank controls its leverage by securitization of loans. The fixed transaction costs in such a securitization lead to a formulation in an impulse control framework. The bank operates in a Markov-switching economy and wants to maximize the utility of terminal equity value. For the quasi-variational inequality associated with this impulse control problem, we prove existence and uniqueness of viscosity solutions. Iterated optimal stopping is used to find a numerical solution of this PDE, and numerical examples are discussed.