Summary of the

DISSERTATION

Scaling laws in two models for thermodynamically driven fluid flow

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In this thesis, we consider two models from physics, which are characterized by the interplay of thermodynamical and fluid mechanical phenomena: demixing (spinodal decomposition) and Rayleigh–Bénard convection. In both models, we investigate the dependencies of certain intrinsic quantities on the system parameters.

The first model describes a thermodynamically driven demixing process of a binary viscous fluid. During the evolution, the two components of the mixture separate into two domains of the different equilibrium volume fractions. One observes a clear tendency: Larger domains grow at the expense of smaller ones, and thus, the average domain sizes increases — a phenomenon called coarsening. It turns out that two mechanisms are relevant for the coarsening process. At an early stage of the evolution, material transport is essentially mediated by diffusion; at a later stage, when the typical domain size exceeds a certain value, due to the viscosity of the mixture, a fluid flow sets in and becomes the relevant transport mechanism. In both regimes, the growth rates of the typical domain size obey certain power laws. In this thesis, we rigorously establish one-sided bounds on these growth rates via a priori estimates.

The second model, Rayleigh–Bénard convection, describes the behavior of a fluid between two rigid horizontal plates that is heated from below and cooled from above. There are two competing heat transfer mechanisms in the system: On the one hand, thermodynamics favors a state in which temperature variations are locally minimized. Thus, in our model, the thermodynamical equilibrium state is realized by a temperature with a linearly decreasing profile, corresponding to pure conduction. On the other hand, due to differences in the densities of hot and cold fluid parcels, buoyancy forces act on the fluid. This results in an upward motion of hot parcels and a downward motion of cold parcels. We study the dependence of the average upward heat flux, measured in the so-called Nusselt number, on the temperature forcing encoded by the container height. It turns out that the efficiency of the heat transport is independent of the height of the container, and thus, the Nusselt number is a constant function of height. Using a priori

estimates, we prove an upper bound on the Nusselt number that displays this dependency — up to logarithmic errors.

Further investigations on the flow pattern in Rayleigh–Bénard convection show a clear separation of length scales: Along the horizontal top and bottom plates one observes thin boundary layers in which heat is essentially conducted, whereas the large bulk is characterized by a convective heat flow. We give first rigorous results in favor of linear temperature profiles in the boundary layers, which indicate that heat is indeed essentially conducted close to the boundaries.