

The Eyring-Kramers formula for Poincaré and logarithmic Sobolev inequalities

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Dissertation

Abstract

The topic of this thesis is a diffusion process on a potential landscape which is given by a smooth Hamiltonian function in the regime of small noise. The work provides a new proof of the Eyring-Kramers formula for the Poincaré inequality of the associated generator of the diffusion. The Poincaré inequality characterizes the spectral gap of the generator and establishes the exponential rate of convergence towards equilibrium in the L^2 -distance. This result was first obtained by Bovier et. al. in 2004 relying on potential theory.

The presented approach in the thesis generalizes to obtain also asymptotic sharp estimates of the constant in the logarithmic Sobolev inequality. The optimal constant in the logarithmic Sobolev inequality characterizes the convergence rate to equilibrium with respect to the relative entropy, which is a stronger distance as the L^2 -distance and slightly weaker than the L^1 -distance. The optimal constant has here no direct spectral representation.

The proof makes use of the scale separation present in the dynamics. The Eyring-Kramers formula follows as a simple corollary from the two main results of the work: The first one shows that the associated Gibbs measure restricted to a basin of attraction has a good Poincaré and logarithmic Sobolev constants providing the fast convergence of the diffusion to metastable states. The second main ingredient is a mean-difference estimate. Here a weighted transportation distance is used. It contains the main contribution to the Poincaré and logarithmic Sobolev constant, resulting from exponential long waiting times of jumps between metastable states of the diffusion.