

# A Search for Finer Topological Information by Holomorphic Curves in Symplectizations: the Case of Lens Spaces and Their Unit Cotangent Bundles

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## Abstract

In this thesis, we use the moduli spaces of punctured holomorphic curves in the symplectizations of 3-dimensional lens spaces and in the symplectizations of their unit cotangent bundles in neck-stretching arguments, in order to regain the well-known diffeomorphism classification of 3-dimensional lens spaces.

In the case of the unit cotangent bundles, we study the contact structures, which are quotients of the contact structure on the unit cotangent bundle of the round 3-dimensional sphere. In the case of lens spaces, we study the contact structures, which are quotients of the standard contact structure on 3-dimensional sphere. In both cases, after choices of symmetric contact forms and suitable almost complex structures, the moduli spaces of certain types of holomorphic curves are explicitly determined.

We concentrate on the moduli space of pair of pants with non contractible positive ends, where for each component of the moduli space the homotopy relation between two positive ends of the pair of pants encodes the defining parameter of the given lens space. Using a neck-stretching argument, we compare these homotopy relations, first under the presence of a contactomorphism between the unit cotangent bundles of two lens spaces and second under the presence of a contactomorphism between two lens spaces.

In the first case, we prove that if the contactomorphism satisfies certain pinching conditions, then defining parameters of the underlying lens spaces satisfy the diffeomorphism classification condition. In the second case, we achieve the same classification condition if the contactomorphism is strict along a single non contractible orbit.