

Lumpability of evolution equations in Banach spaces

Abstract

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In this thesis we analyze *lumpability* of infinite dimensional dynamical systems. *Lumping* is a method to project a dynamics by a linear reduction operator onto a smaller state space on which a self-contained dynamical description exists. We consider the system $\dot{x} = F(x)$ defined on a Banach space X , together with a linear and bounded map $M : X \rightarrow Y$, where Y is another Banach space. The operator M is surjective but not an isomorphism and it represents a *reduction* of the state space. We investigate whether the variable $y = Mx$ also satisfies a well-posed and self-contained dynamics on Y . This happens if and only if an operator \hat{F} exists on Y in such a way that $MF = \hat{F}M$ holds, and the system $\dot{y} = \hat{F}(y)$ admits a unique solution for every initial condition. If possible, we implicitly define the reduced operator by $\hat{F}(y) := MF(x)$, for $y = Mx$. We first discuss lumpability of linear systems in Banach spaces. In this context, we assume that F generates a C_0 -semigroup $T(t)$ on X . We give conditions for the reduced operator to exist and to be itself the infinitesimal generator of a reduced C_0 -semigroup on Y . We extend these results to the dual Banach space, and we describe the behaviour of the adjoint operator M^* in relation to a particular subspace of X^* , called the *sun dual space*. Next, we study lumpability of nonlinear evolution equations. We focus on semigroups of contractions, for which some interesting results exist, concerning the existence and uniqueness of solutions, both in the classical sense of smooth solutions and in the weaker sense of *strong solutions*. It is known that, under suitable hypotheses (e.g. dissipativity) F generates a semigroup of nonlinear contractions in the sense of Crandall-Liggett: F is not necessarily the infinitesimal generator of $T(t)$, but if the semigroup is differentiable for almost every $t \geq 0$, then it is the unique solution of the Cauchy problem $\dot{x} = F(x)$. We discuss in details under which conditions the operator \hat{F} can be again associated with a nonlinear, strongly continuous semigroup giving the solutions of the reduced system. We also investigate the regularity properties inherited by \hat{F} from the original operator F . Finally, we describe a particular kind of lumping in the context of C^* -algebras. This lumping represents a different interpretation of the restriction operator from $C_0(S)$ to $C_0(\mathcal{C})$, S and \mathcal{C} being a locally compact, Hausdorff space and a closed subset, respectively. We apply this lumping to *Feller semigroups*, which are important because they can be associated in a unique way to Markov processes. We show that the fundamental properties of Feller semigroups are preserved by this lumping. Using these ideas, we give a short proof of the classical Tietze extension theorem based on C^* -algebras and Gelfand theory.