Twistor Spinors on Riemannian Spin Orbifolds

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Abstract

Twistor spinors are solutions of a conformal covariant field equation for spinor fields. The motivation for this equation originally came from mathematical physics where this equation was introduced by R. Penrose in 1967. In this thesis, we examine the behaviour of the length function of a twistor spinor with zero on a Riemannian spin orbifold with isolated singularities near its zero. In the orbifold setting, these special spinors are considered locally as G_p-equivariant twistor spinors on a fundamental chart, where $G_p \subset Spin(n)$ is the isomorphic pre-image of the local isotropy group $\Gamma_p \subset SO(n)$ of a point p on the spin orbifold. To write the Taylor expansion of the length function, we use a conformal normal coordinate system which is a Riemannian normal coordinate system for a particular metric in the conformal class. We choose actually a conformal normal coordinate system that provides $\det g = 1$ up to an order at least 5. This is a special case of Lee and Parker's conformal normal coordinates. This property gives a simplification in the expansion. Hence the length of the twistor spinor is written up to order 7. Schouten tensor plays an important role in the expansion. We use this development for a twistor spinor with zero on an Eguchi-Hanson orbifold which is a conformal inversion of Eguchi-Hanson manifold. Eguchi-Hanson manifold is an asymptotically locally Euclidean manifold of order 4 and a spin manifold admitting a 2-dimensional space of parallel spinors. For this explicit example, we find a conformal normal coordinate system which guarantees $\det g = 1 + O(r^8)$, where r is the distance function in Riemannian normal coordinates. The length of the twistor spinor is written in this system up to order 10. As a second application of the development of the length function, we compute the mass of an asymptotically locally Euclidean system obtained from the conformal normal coordinate system around the zero of a twistor spinor on a Riemannian spin orbifold with isolated singularities. The mass that we consider is a generalization of the ADM mass introduced for an isolated gravitational system by Arnowitt, Deser and Misner in 1961. The mass of an asymptotic system is actually an invariant of the metric under some asymptotic decay conditions. It is given by the limit of a certain integral over the spheres in the end. As the isolated singular points on an orientable Riemannian orbifold occur in even dimensions, our result for the mass is for even dimensions. The order of the metric in conformal normal coordinates is 3. After a conformal inversion, the asymptotic decay of the metric is found as 3. Thus, the decay of the constructed asymptotic system on the end guarantees a finite mass for dimensions 4 and 6. The computations show that the mass is actually zero in these dimensions.