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In the first chapter we recall the classical time discretization scheme by Almgren-Taylor-Wang and Luckhaus-Sturzenhecker. After a short interlude on signed distance functions the scheme is adapted according to the necessities of our parabolic obstacle problem.
The remainder of the first chapter is dedicated to the analysis of this adapted scheme. We prove existence and graphicality of minimizers of the elliptic problems and motivate a reformulation of the scheme in the class of Lipschitz functions. Finally we show that this new formulation yields the same solutions.

The study of a class of obstacle problems, which encompasses the minimization problems arising from the reformulated scheme is the content of the second chapter.
We are employing an approach introduced by Hartman and Stampacchia which transforms these problems – which are instances of (elliptic) variational inequalities – to a problem on an even smaller class of functions and eventually reduces to the construction of so called barriers, a tool to deduce a priori boundary gradient estimates.
Although this is just one possible way of deducing the existence of Lipschitz solutions of the time-independent elliptic problems – another one is also discussed – this one allows us to derive suitable bounds on the Lipschitz constant. This argument can then be iterated in the next chapter.
We then also discuss the higher regularity of solutions to certain obstacle problems without using the usual penalisation arguments.
Eventually, we use the results from this chapter to derive existence, uniqueness and regularity of constrained minimal graphs together with a useful characterisation of them.

Chapter three begins with a detailed explanation how the results of the second chapter can be used to derive the existence of approximate flows with uniform control on the (spacial-) Lipschitz constant. Subsequently, this Lipschitz bound is heavily used to derive a number of important properties of this time discrete evolution. These results can then be used to pass into the limit in the time step size to obtain so called flat flows.
After deriving L^2-spacetime bounds on the derivatives of these flat flows, we show that they are indeed solving the variational inequality and can thus also be called distributional solutions.
A final section deals with the asymptotic limit as time goes to infinity. We show that flat flows converge uniformly to constrained minimal graphs.

In the final chapter, we first of all introduce yet another way to formulate our parabolic obstacle problem, namely the notion of viscosity solutions. This concept is particularly flexible as it requires the solutions merely to be continuous. Then we show that the flat flows obtained in chapter three are also viscosity solutions.
Having established this connection, we deduce a variant of the classical comparison principle for viscosity sub- and supersolutions of the parabolic obstacle problem which not only gives us another characterisation of flat flows but also a way to deduce uniqueness of flat flows and thus to remove the requirement to pass to subsequences of approximate flows.