SIMPLE MODULAR LIE SUPERALGEBRAS (ABSTRACT)

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My work is a contribution to the classification of simple modular Lie algebras and superalgebras. In particular, it is a step in support of a super-analog of Kostrikin-Shafarevich conjecture (recently formulated by D. Leites) that describes all simple finite dimensional modular Lie superalgebras over algebraically closed fields.

Generalized versions of the original Kostrikin-Shafarevich conjecture concerning restricted and non-restricted Lie algebras over algebraically closed fields of characteristic $p > 3$ have been proven by Block, Wilson, Premet and Strade. The situation with $p = 3$ and 2 and with Lie superalgebras is still unclear.

A notable contribution to its clarification is due to A. Elduque, who offered a new approach to the Freudenthal Magic Square thus giving an interpretation to nine new simple finite dimensional Lie superalgebras in characteristic 3 (and found one in characteristic 5).

Another landmark is the latest (yet unpublished) result due to Bouarroudj, Grozman, and Leites: classification of all simple finite dimensional Lie superalgebras with Cartan matrix over algebraically closed fields of any positive characteristic.

My work is devoted to description of some of Lie algebras and superalgebras over algebraically closed (or, in some cases, perfect) fields of characteristic 2. Here is a brief summary of the results:

• Non-degenerate bilinear forms over fields of characteristic 2, in particular, non-symmetric ones, are classified with respect to various equivalences, and the Lie algebras preserving them are described. Several types of equivalences of bilinear forms are investigated and one of these equivalences (considered by Albert in 1940s) is shown to be related with contact differential 1-forms (on manifolds and supermanifolds).

Although it was known that there are two series of non-isomorphic finite simple Chevalley groups preserving the non-degenerate symmetric bilinear forms on the space of even dimension, the description of simple Lie algebras that preserve these forms seems to be new.

• The analogs of the above results for superspaces are also given. Non-degenerate bilinear forms over fields of characteristic 2, both supersymmetric and non-supersymmetric ones, are classified with respect to various equivalences, and the Lie superalgebras preserving them are described. The fact that the periplectic Lie superalgebra (or rather its nontrivial central extension existing only for $p = 2$) possesses for $p = 2$ a Cartan matrix was unexpected.

• Analogs of the Hamiltonian and Poisson Lie superalgebras over a perfect field of characteristic 2 are described. Unlike characteristic 0 case, there are several such analogs. Characteristic 2 analogs of the antibracket and related Lie superalgebras are also described. The existence of several types of the Poisson bracket and the impossibility to interpret some of the associated Lie algebras of Hamiltonian vector fields as preserving a differential 2-form (even understood in terms of divided powers) are unexpected.

• The defining relations (of both Serre and non-Serre types) for Chevalley generators are listed for the simple modular Lie algebras with Cartan matrix and for the simple Lie algebras obtained from the complex Lie algebras with Cartan matrix after reduction of the structure constants modulo p but having no Cartan matrix after the reduction.

• For the modular Lie (super)algebras of the form $\mathfrak{g}(A)$, the modular analog of the reflections which permute the systems of simple roots is described.

• Two new series of simple finite-dimensional Lie superalgebras (relatives of $\mathfrak{q}(\mathfrak{o}_I(n))$ and $\mathfrak{q}(\mathfrak{o}_\Pi(2n))$) and three exceptional simple finite-dimensional Lie superalgebras ($\mathfrak{q}(\mathfrak{e}(6))$, $\mathfrak{q}(\mathfrak{e}^{(1)}(7)/\mathfrak{center})$, $\mathfrak{q}(\mathfrak{e}(8))$) were discovered.

• All nonequivalent systems of simple roots for the seven new $\mathfrak{e}$-type Lie superalgebras $\mathfrak{e}(a, b)$ discovered by Bouarroudj, Grozman and Leites were classified.