

Abstract of the dissertation
“Analysis of boundary vortices in
thin magnetic films”

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In this thesis we analyze the asymptotic behavior as $\varepsilon \rightarrow 0$ of the functionals

$$E_\varepsilon(u) = \frac{1}{2} \int_\Omega |\nabla u|^2 dx + \frac{1}{2\varepsilon} \int_{\partial\Omega} \sin^2(u - g) d\mathcal{H}^1,$$

where Ω is a simply connected domain in \mathbb{R}^2 and $g : \partial\Omega \rightarrow \mathbb{R}$ is a function such that $e^{ig} : \partial\Omega \rightarrow S^1$ is a map of degree $d \neq 0$.

These functionals correspond to a certain thin-film limit of the micromagnetic energy derived by Kohn and Slastikov. The limit $\varepsilon \rightarrow 0$ leads to the formation of boundary vortices that can be seen as a boundary analogue of the interior Ginzburg-Landau vortices studied by Bethuel-Brezis-Hélein. Similar to their work, we derive convergence results for minimizers and also for critical points of the energy that satisfy a natural energy bound. One of the key steps in the proof is a localization result for the vortices that is based on the approach of Struwe to Ginzburg-Landau vortices and relies on a Rellich-Pohožaev identity.

We further show that the energy asymptotically behaves like the sum of a singular term depending only on the number of vortices and an order 1 term that depends only on the vortex positions, the *renormalized energy*.

Also, we show that a natural rescaling of the energy functional restricted to the boundary Γ -converges to a functional that only counts the number of vortices. This generalizes a result of Alberti-Bouchitté-Seppecher for the coercive two-well case to that of a periodic potential.

Finally, we study what happens if the domain has corners. Arguing again with the renormalized energy allows us to show that the “S” state of a rectangular thin magnetic sample is within the validity of our approximations energetically more favorable than the “C” state.