

Krähmer, Ulrich Arthur:

On Quantum Double Groups and Quantum Flag Manifolds. Two Mathematical Structures with Possible Applications in q -Deformed Field Theories.

6+56 pages, 101 references.

PhD thesis, Universität Leipzig, 2004

SUMMARY

The overall topic of this PhD thesis is the theory of quantum groups and their homogeneous spaces.

In the first part of the thesis, l -functionals on coquasitriangular Hopf algebras are studied. First, a formula from [KS] relating the l -functionals on the standard quantum group coordinate algebra $\mathbb{C}_q[SL(N+1, \mathbb{C})]$ to Lusztig's quantum root vectors in $U_q(\mathfrak{sl}(N+1, \mathbb{C}))$ is generalized to the other classical matrix Lie groups. It allows to substitute the root vectors in the Poincaré-Birkhoff-Witt basis of $U_q(\mathfrak{g})$ by l -functionals. Then one has explicit formulas for the commutation relations and the coproduct of the basis elements. Afterwards, the Hopf algebra generated by the l -functionals on the quantum double $\mathbb{C}_q[G] \bowtie \mathbb{C}_q[G]$ is shown to be isomorphic to $\mathbb{C}_q[G]^{\text{op}} \bowtie U_q(\mathfrak{g})$ for all semisimple G . This was conjectured by T. Hodges in [Ho]. As an algebra, $\mathbb{C}_q[G]^{\text{op}} \bowtie U_q(\mathfrak{g})$ can be embedded into $U_q(\mathfrak{g} \oplus \mathfrak{g})$, see [Ho]. Here it is proven that there is no bialgebra structure on $U_q(\mathfrak{g} \oplus \mathfrak{g})$ for which this embedding becomes a homomorphism of bialgebras. In particular, it is not an isomorphism.

The second part deals with the theory of covariant differential calculi on quantum homogeneous spaces, and with their relation to A. Connes' non-commutative geometry. A Dirac operator on the standard quantization of the compact Hermitian symmetric spaces is defined. This yields a Hilbert space realization of the covariant differential calculi constructed by I. Heckenberger and S. Kolb in [HK]. All differentials $\mathfrak{f} = i[D, f]$ are bounded operators. In the simplest case of Podleś' standard quantum sphere one obtains the spectral triple found by L. Dąbrowski and A. Sitarz [DS].

In a short third part we prove that in contrast to their C^* -completions, the coordinate algebras of the non-standard Podleś spheres depend on the additional deformation parameter. This was posed as an open problem in [HMS].

The results of the first two parts of this work are published in [Kr1, Kr2].

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