

The Super Conformal Action Functional on Super Riemann Surfaces

Zusammenfassung

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The super conformal action functional appeared in the context of string theory in the 1970s as a two-dimensional non-linear super symmetric sigma model. Its main interest lies in its many symmetries, in particular super symmetry. Because super symmetry requires anti-commutative functions, ordinary differential geometry cannot be sufficient to give a unified geometric treatment of the super conformal action functional. Instead, this thesis shows that all the symmetries of the super conformal action functional obtain a natural geometric interpretation in the language of families of super manifolds.

Super manifolds, introduced in the 1980s, are generalizations of manifolds with additional odd dimensions which are realized by additional Grassmann variables in the structure sheaf. The first part of this thesis treats the super differential geometry of families of super manifolds. Appropriate generalizations of principal bundles, smooth families of complex manifolds and integration theory are developed. A phenomenon that is unique to families of super manifolds is the underlying even manifold. An underlying even manifold to a family of super manifolds M is an embedded family $|M|$ with odd fiber dimension zero. It is shown that the family $|M|$ is unique, but different such embeddings exist.

The second part of this thesis treats super Riemann surfaces. Super Riemann surfaces are particular complex super manifolds of dimension $1|1$. It is shown that a super Riemann surface is completely determined by a metric, spin structure and gravitino field on the underlying even manifold. The super conformal action functional is given as a Berezin integral over the super Riemann surface that has remarkable similarities to the harmonic action functional on Riemann surfaces. Super symmetry of the super conformal action functional is now seen to be super diffeomorphism invariance of the action functional. By its symmetries, the super conformal action functional can be interpreted as a functional on the moduli space of super Riemann surfaces. The fact that it can be consistently formulated for a non-linear target space and the analogy to the case of Riemann surfaces leads to the hope that the super conformal action functional might allow to study the moduli space of super Riemann surfaces with methods from geometric analysis. As a first application the space of infinitesimal deformations of a super Riemann surface is calculated in terms of metric and gravitino.