This work addresses the question of the efficient numerical solution of time-domain boundary integral equations with retarded potentials arising in the problems of acoustic and electromagnetic scattering. The convolutional form of the time-domain boundary operators allows to discretize them with the help of Runge-Kutta convolution quadrature. This method combines Laplace-transform and time-stepping approaches and requires the explicit form of the fundamental solution only in the Laplace domain to be known. Recent numerical and analytical studies revealed excellent properties of Runge-Kutta convolution quadrature, e.g. high convergence order, stability, low dissipation and dispersion.

As a model problem, we consider the wave scattering in three dimensions. The convolution quadrature discretization of the indirect formulation for the three-dimensional wave equation leads to the lower triangular Toeplitz system of equations. Each entry of this system is a boundary integral operator with a kernel defined by convolution quadrature. In this work we develop an efficient method of almost linear complexity for the solution of this system based on the existing recursive algorithm. The latter requires the construction of many discretizations of the Helmholtz boundary single layer operator for a wide range of complex wavenumbers. This leads to two main problems: the need to construct many dense matrices and to evaluate many singular and near-singular integrals.

The first problem is overcome by the use of data-sparse techniques, namely, the high-frequency fast multipole method (HF FMM) and $\mathcal{H}$-matrices. The applicability of both techniques for the discretization of the Helmholtz boundary single-layer operators with complex wavenumbers is analyzed. It is shown that the presence of decay can favorably affect the length of the fast multipole expansions and thus reduce the matrix-vector multiplication times. The performance of $\mathcal{H}$-matrices and the HF FMM is compared for a range of complex wavenumbers, and the strategy to choose between two techniques is suggested.

The second problem, namely, the assembly of many singular and nearly-singular integrals, is solved by the use of the Huygens principle. In this work we prove that kernels of the boundary integral operators $w_n^h(d)$ (h is the time step and $t_n = nh$ is the time) exhibit exponential decay outside of the neighborhood of $d \approx nh$ (this is the consequence of the Huygens principle). The size of the support of these kernels for fixed h increases with n as n^α, $\alpha < 1$, where α depends on the order of the Runge-Kutta method and is (typically) smaller for Runge-Kutta methods of higher order. Numerical experiments demonstrate that theoretically predicted values of α are quite close to optimal.

In the work it is shown how this property can be used in the recursive algorithm to construct only a few matrices with the near-field, while for the rest of the matrices the far-field only is assembled. The resulting method allows to solve the three-dimensional wave scattering problem with asymptotically almost linear complexity. The efficiency of the approach is confirmed by extensive numerical experiments.