Local Prime Factor Decomposition of Approximate Strong Product Graphs

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Graphs and in particular graph products arise in a variety of different contexts, from computer science to theoretical biology, computational engineering or in studies on social networks.

In practice, we observe perturbed product structures, so-called *approximate* graph products, since structures derived from real-life data are notoriously incomplete and/or plagued by measurement errors. In fact, a very small perturbation, such as a deletion or insertion of a single edge, can destroy the product structure completely, modifying a product graph to a prime graph. The practical application of the well-known prime factorization algorithms is therefore limited, since most graphs are prime, although they can have a product-like structure. In order to deal with such inaccuracies, a mathematical framework is needed that allows us to deal with graphs that are only approximate products.

In this thesis, we are in particular interested in the so-called *strong graph product*, that is one of the four standard products. Since strong product graphs G contain subgraphs that are itself products of subgraphs of the underlying factors of G, we follow the idea to develop local approaches that cover a graph by factorizable patches and then use this information to derive the global factors.

First, we investigate the local structure of strong product graphs and introduce the *backbone* $\mathbb{B}(G)$ of a graph G and the so-called *S1-condition*. Both concepts play a central role for determining the prime factors of a strong product graph in a unique way. Then, we discuss several graph classes, in detail, *NICE*, *CHIC* and *locally unrefined* graphs. For each class we construct local, quasi-linear time prime factorization algorithms. Combining these results, we then derive a new local, quasi-linear time prime factorization algorithm for all graphs.

Finally, we discuss approximate graph products. We use the new local factorization algorithm to derive a method for the recognition of approximate graph products. Furthermore, we evaluate the performance of this algorithm on a sample of approximate graph products.