

# Green's function estimates for elliptic and parabolic operators: Applications to quantitative stochastic homogenization and invariance principles for degenerate random environments and interacting particle systems

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## Abstract.

This thesis is divided into two parts: In the first one (Chapters 1 and 2), we deal with problems arising from quantitative homogenization of the random elliptic operator in divergence form  $-\nabla \cdot a \nabla$ . In Chapter 1 we study existence and stochastic bounds for the Green function  $G$  associated to  $-\nabla \cdot a \nabla$  in the case of systems. Without assuming any regularity on the coefficient field  $a = a(x)$ , we prove that for every (measurable) uniformly elliptic tensor field  $a$  and for almost every point  $y \in \mathbb{R}^d$ , there exists a unique Green's function centred in  $y$  associated to the vectorial operator  $-\nabla \cdot a \nabla$  in  $\mathbb{R}^d$ ,  $d > 2$ . In addition, we prove that if we introduce a shift-invariant ensemble  $\langle \cdot \rangle$  over the set of uniformly elliptic tensor fields, then  $\nabla G$  and its mixed derivatives  $\nabla \nabla G$  satisfy optimal pointwise  $L^1$ -bounds in probability.

Chapter 2 deals with the homogenization of  $-\nabla \cdot a \nabla$  to  $-\nabla a_{\text{hom}} \nabla$  in the sense that we study the large-scale behaviour of  $a$ -harmonic functions in exterior domains  $\{|x| > r\}$  by comparing them with functions which are  $a_{\text{hom}}$ -harmonic. More precisely, we make use of the first and second-order correctors to compare an  $a$ -harmonic function  $u$  to the two-scale expansion of suitable  $a_{\text{hom}}$ -harmonic function  $u_h$ . We show that there is a direct correspondence between the rate of the sublinear growth of the correctors and the smallness of the relative homogenization error  $u - u_h$ .

The theory of stochastic homogenization of elliptic operators admits an equivalent probabilistic counterpart, which follows from the link between parabolic equations with elliptic operators in divergence form and random walks. This allows to reformulate the problem of homogenization in terms of invariance principle for random walks. The second part of thesis (Chapters 3 and 4) focusses on this interplay between probabilistic and analytic approaches and aims at exploiting it to study invariance principles in the case of degenerate random conductance models and systems of interacting particles.

In Chapter 3 we study a random conductance model where we assume that the conductances are independent, stationary and bounded from above but not uniformly away from 0. We give a simple necessary and sufficient condition for the relaxation of the environment seen by the particle to be diffusive in the sense of every polynomial moment. As a consequence, we derive polynomial moment estimates on the corrector which imply that the discrete elliptic operator homogenises or, equivalently, that the random conductance model satisfies a quenched invariance principle.

In Chapter 4 we turn to a more complicated model, namely the symmetric exclusion process. We show a diffusive upper bound on the transition probability of a tagged particle in this process. The proof relies on optimal spectral gap estimates for the dynamics in finite volume, which are of independent interest. We also show off-diagonal estimates of Carne-Varopoulos type.