Efficient Update of Hierarchical Matrices in the case of Adaptive Discretisation Schemes

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Abstract

Scientific computing is a research field that arises from numerical mathematics and has as a major task the efficient implementation of numerical algorithms. This work represents a small contribution to this field.

Very popular and widely used methods of solving integral equations numerically are discretisation schemes like Ritz-Galerkin or collocation techniques. These methods solve the integral equation approximately by solving a system of linear equations. The problem that arises is that the matrix of the system of linear equations is, in the general case, densely populated. In order to overcome this obstacle we can approximate that matrix using methods like hierarchical matrices which will be our choice in this work.

The hierarchical matrix technique (or briefly $\mathcal{H}$-matrix technique) has been developed during the past ten years. The main property of the hierarchical matrices is their **data-sparse** structure and the main advantage is that $\mathcal{H}$-matrix arithmetic can be performed in almost optimal complexity $\mathcal{O}(n \log^c n)$ for $n \times n$ systems and small integer c. The $\mathcal{H}$-matrix technique has been efficiently applied in many research fields (FEM, BEM, control theory).

The central part of this work is the update of hierarchical matrices. The motivation for the update of $\mathcal{H}$-matrices comes from two directions. One of them is the discretisation and on the other hand it is the $\mathcal{H}$-matrix itself. A discretisation can become finer in different ways but an especially interesting one is when it is refined only locally. More precisely, if the new discretisation contains, compared to the old discretisation, only few new elements, solving the system of equations will demand the full assembly of an $\mathcal{H}$-matrix. Assembling the $\mathcal{H}$-matrix is not just memory but also time-consuming. Therefore we try to improve both time and storage requirements. If we assume that the $\mathcal{H}$-matrix corresponding to the old discretisation exists, in order to obtain the $\mathcal{H}$-matrix corresponding to the new discretisation, we choose an alternative way which includes recycling the "old" $\mathcal{H}$-matrix. This method we will call **update of hierarchical matrices**. The description of this method and testing its efficiency are the central parts of this work. The update of an $\mathcal{H}$-matrix is considerably faster than constructing a new one.

The thesis is divided into seven chapters. The first chapter contains the precise definition of the problem we try to solve and some general properties of the boundary element method (BEM). Chapter 2 contains the definition of the cluster tree, several clustering algorithms and the definition of the block cluster tree. The third chapter contains the definition of the hierarchical matrices and algorithms for obtaining the low-rank approximation. In the last section of this chapter we explain the motivation for the update and the purpose of the update algorithm. The update algorithm is explained in two separate chapters. Chapter 4 will consider the update of the cluster tree, while Chapter 5 explains the update of hierarchical matrices. Chapter 6 contains the numerical tests of our method and the last Chapter is devoted to implementational details.