Large Deviations Results for Likelihood Ratios and Hypothesis Testing on Quantum Spin Chains

Yuri Cassio Campbell Borges[1]

[1] Max Planck Institute for Mathematics in the Sciences, Inselstr. 22, 04103 Leipzig, Germany
campbell@mis.mpg.de

Abstract

We consider a formally natural extension of the Chernoff distance between two probability measures on a finite set to the range of sequences of such measures. If both sequences represent joint distributions of respective i.i.d. stochastic processes with a finite state space, due to a famous result by Chernoff [1], the Chernoff distance of the corresponding laws represents a sharp bound on the exponential decay rate of the Bayesian error probability in asymptotic symmetric hypothesis testing. This generalization of the Chernoff distance has appeared first in the work by Hiai et al. [2] in the context of sequences of density operators on finite-dimensional complex Hilbert spaces, which are associated with states on quantum spin chains. By means of a mapping introduced in [3], the Pinching Map, a pair of sequences of density operators gives rise to a pair of sequences of probability measures on a sequence of finite sets, and the associated generalized Chernoff distances of both pairs coincide. We show that hypothesis testing between two sequences of density matrices associated to finite dimensional quantum states is asymptotically equivalent to a corresponding testing problem between two sequences of probability measures and provide asymptotic lower and upper bounds on the exponential decay rate of the associated Bayesian error probability. As our main result we specify necessary and sufficient conditions under which the bounds coincide with the generalized Chernoff distance. Thereby we unify both frameworks and provide the operational meaning of this extended concept.

keywords: Chernoff bound, asymptotic symmetric hypothesis testing, Neyman-Pearson lemma, maximum likelihood tests, large deviations applications.

References

[1] H. Chernoff, “A measure of asymptotic efficiency for tests of a hypothesis based on the sum of observations,” *The Annals of Mathematical Statistics*, vol. 23, no. 4, pp. 493–507, Dec. 1952.

[2] F. Hiai, M. Mosonyi, and T. Ogawa, “Error exponents in hypothesis testing for correlated states on a spin chain,” *Journal of Mathematical Physics*, vol. 49, no. 3, p. 032112, Mar. 2008.

[3] M. Nussbaum and A. Szkoła, “The Chernoff lower bound for symmetric quantum hypothesis testing,” *The Annals of Statistics*, vol. 37, no. 2, pp. 1040–1057, Apr. 2009.