On the Construction and Behaviour of Automorphic Forms on Completions of Teichmüller Spaces and Higher Bers Maps

Thesis Summary

As the title suggests, this thesis consists of two rather separate bodies of work. Namely, we investigate the construction and behaviour of automorphic forms over families of Riemann surfaces and we introduce the theory of higher Bers maps.

Higher Bers Maps. The Teichmüller space Teich(Σ) of a Riemann surface of finite type Σ, which can be written as $\Sigma = \mathbb{D}/G$, is the universal object in the category of families of marked Riemann surfaces, and it admits several natural representations: One can either represent Teichmüller space by equivalence classes of measurable functions on the unit disc satisfying an automorphy condition (the *Beltrami model*) or by a subset of the schlicht functions of the unit disc (the *schlicht model*).

Using non-linear differential operators acting on schlicht functions, we proved a general theorem on when such operators induce holomorphic mappings of Teichmüller space into complex Banach spaces, and furthermore gave examples of such operators. Two series, the A- and B- higher Schwarzians, which induce holomorphic mappings we dubbed *higher Bers maps* due to their similarity to the Bers embedding, were studied in great detail. In particular, their infinitesimal behaviour was fuly established. We also pointed out future questions and applications of these mappings.

Construction and Behaviour of Families of Automorphic Forms. Motivated by the classical theory of harmonic mappings of Riemann surfaces into other manifolds and their behaviour under changes of the conformal structure of the domain surface, in particular under its degeneration, we provided an explicit analytic construction of the universal object in the category of families of marked Riemann surfaces equipped with holomorphic line bundles in terms of s-factor of automorphy. Further we showed that this family always has holomorphic sections and gave an explicit algorithm on how to construct these starting from a holomorphic function on the unit disc. We show that the fibre space so obtained is a complex vector bundle $\mathcal{V}(\mathcal{R})$ with a smooth hermitian structure and that the previous method of extension yields a holomorphic trivialization almost everywhere.

Finally we show that the vector bundle $\mathcal{V}(\mathcal{R})$ can be extended to a fibre space $\hat{\mathcal{V}}(\mathcal{R})$ over augmented Teichmüller space $\hat{T}(G)$ and that the previously constucted sections extend to continuous sections of $\hat{\mathcal{V}}(R)$. This result also gives a complete answer to the question above concerning the geometric nature of the limit point in terms of a noded Riemann surface and a collection of line bundles on parts of the surface.

Applications. There are two appendices to the thesis, which indicate two different applications where the explicit construction of families of automorphic forms becomes important. The first area of application is the broad subject of (super)conformal σ-models with auxiliary fields given by sections of holomorphic line bundles (possibly twisted with the pull back of a bundle over the target); particular examples are the model of *Dirac-harmonic maps* and the equations governing *Supersymmetric String Theory*. In Appendix we sketch how and why this work is related to problems in such models.

The second application concerns a problem motivated by the theory of geometric quantization, for which one needs the Chern connection of the hermitian bundle $\mathcal{V}(\mathcal{R})$. We indicate that this connection is computable in the frame of sections constructed previously due to the explicit way the sections vary over Teichmüller space.