Onsager's Conjecture

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In 1949, Lars Onsager in his famous note on statistical hydrodynamics conjectured that weak solutions to the 3-D incompressible Euler equations belonging to Hölder spaces with Hölder exponent greater than 1/3 conserve kinetic energy; conversely, he conjectured the existence of solutions belonging to any Hölder space with exponent less
than 1/3 which do not conserve kinetic energy. The first part, relating to conservation of kinetic energy, has since been confirmed (cf. Eyink 1994, Constantin-E-Titi 1994). The second part, relating to the existence of non-conservative solutions, remains an open conjecture and
is the subject of this dissertation.

In groundbreaking work of De Lellis and Székelyhidi Jr. (2012), the
authors constructed the first examples of non-conservative Hölder continuous weak solutions to the Euler equations. The construction was subsequently improved by Isett (2012/2013), introducing many novel ideas in order to construct $1/5^-$ Hölder continuous weak solutions with compact support in time.

Adhering more closely to the original scheme of De Lellis and Székelyhidi Jr., we present a comparatively simpler construction of $1/5^-$ Hölder continuous non-conservative weak solutions which may in addition be made to obey a prescribed kinetic energy profile. Furthermore, we extend this scheme in order to construct weak non-conservative solutions to the Euler equations whose Hölder $1/3^-$ norm is Lebesgue integrable in time.

The dissertation will be primarily based on three papers, two of which being in collaboration with De Lellis and Székelyhidi Jr.