Summary

This thesis comprises the study of two moduli spaces of piecewise J-holomorphic curves. The main scheme is to consider a subdivision of the 2-sphere into a collection of small domains and to study collections of J-holomorphic maps into a symplectic manifold. These maps are coupled by Lagrangian boundary conditions. The work can be seen as finding a 2-dimensional analogue of the finite-dimensional path space approximation by piecewise geodesics on a Riemannian manifold (Q, g).

For a nice class of target manifolds we consider tangent bundles of Riemannian manifolds and symplectizations of unit tangent bundles. Via polarization they provide a rich set of Lagrangians which can be used to define appropriate boundary value problems for the J-holomorphic pieces. The work focuses on existence theory as a pre-stage to global questions such as combinatorial refinement and the quality of the approximation.

The first moduli space of lifted type is defined on a triangulation of the 2-sphere and consists of disks in the tangent bundle whose boundary projects onto geodesic triangles. The second moduli space of punctured type is defined on a circle packing domain and consists of boundary punctured disks in the symplectization of the unit tangent bundle. Their boundary components map into single fibers and at punctures the disks converge to geodesics. The coupling boundary conditions are chosen such that the piecewise problem always is Fredholm of index zero and both moduli spaces only depend on discrete data.

For both spaces existence results are established for the J-holomorphic pieces which hold true on a small scale. Each proof employs a version of the implicit function theorem in a different setting. Here the argument for the moduli space of punctured type is more subtle. It rests on a connection to tropical geometry discovered by T. Ekholm for 1-jet spaces. The boundary punctured disks are constructed in the vicinity of explicit Morse flow trees which correspond to the limiting objects under degeneration of the boundary condition.