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Fast Evaluation of Near-Field Boundary Integrals using Tensor Approximations

In this dissertation, we introduce and analyse a scheme for the fast evaluation of integrals stemming from boundary element methods including discretisations of the classical single and double layer potential operators. Our method is based on the parametrisation of boundary elements in terms of a d -dimensional parameter tuple. We interpret the integral as a real-valued function f depending on d parameters and show that f is smooth in a d -dimensional box. A standard interpolation of f by polynomials leads to a d -dimensional tensor which is given by the values of f at the interpolation points. This tensor may be approximated in a low rank tensor format like the canonical format or the hierarchical format. The tensor approximation has to be done only once and allows us to evaluate interpolants in $\mathcal{O}(dr(m+1))$ operations in the canonical format, or $\mathcal{O}(dk^3 + dk(m+1))$ operations in the hierarchical format, where m denotes the interpolation order and the ranks r, k are small integers. In particular, we apply an efficient black box scheme in the hierarchical tensor format in order to adaptively approximate tensors even in high dimensions d with a prescribed (but heuristic) target accuracy. By means of detailed numerical experiments, we demonstrate that highly accurate integral values can be obtained at very moderate costs.