A Priori Bounds for Quasi-Linear SPDEs in the Full Sub-Critical Regime

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In my talk I will be concerned with the quasi-linear parabolic partial differential equation

\[ \partial_t u - a(u) \Delta u = \xi, \]  

(1)

where \( u = u(t,x) \) for \( (t,x) \in \mathbb{R} \times \mathbb{R}^d \), \( \Delta = \sum_{i=1}^d \partial_{x_i}^2 \), and the coefficient field \( u \mapsto a(u) \) is sufficiently smooth and uniformly elliptic. In line with the pathwise approach to stochastic analysis of Lyons, the external forcing \( \xi \) is deterministic and viewed as a realization of a singular noise which a.s. belongs to the (negative) parabolic Hölder space \( C^{\alpha-2} \). For \( \alpha \in (0,\infty) \), the PDE (1) is sub-critical in the sense of Hairer, but in the regime \( \alpha \in (0,1) \) it is not expected to be well-posed in the traditional PDE sense and a re-centering will be needed for the non-linearity \( a(u) \Delta u \), which amounts to adjusting the equation (1) with certain counter-terms, known as a renormalization.

I will present a framework that applies to all sub-critical regularities \( \alpha > 0 \) and all space dimensions \( d \). The input for our theory is a structural assumption on the forcing, which amounts to assuming that various multi-linear functionals of the ‘noise’ \( \xi \) have already been renormalized in an “off-line” probabilistic step. The output is a local Hölder \( a \text{ priori} \) estimate on solutions \( u \) to the renormalized version of (1). We emphasize that the renormalization terms are local in \( u \) and can be constructed explicitly in terms of derivatives of the non-linearity \( a \) and partial information on \( \xi \).

The talk is based on a joint work with Felix Otto, Scott Smith and Hendrik Weber.