Let $K$ be a commutative ring. Consider the groups $\text{GL}_n(K)$. Bernstein and Zelevinsky have studied the representations of these groups in case the ring $K$ is a finite field. Instead of studying the representations of $\text{GL}_n(K)$ for each $n$ separately, they have studied all the representations of all the groups $\text{GL}_n(K)$ simultaneously. They considered on $R := \oplus_n R(\text{GL}_n(K))$ structures called parabolic (or Harish-Chandra) induction and restriction, and showed that they enrich $R$ with a structure they coined positive self adjoint Hopf algebra (or PSH algebra). They use this structure to reduce the study of representations of the groups $\text{GL}_n(K)$ to the following two tasks:

1. Study a special family of representations of $\text{GL}_n(K)$, called “cuspidal representations”. These are representations which do not arise as direct summands of parabolic induction of smaller representations.

2. Study representations of the symmetric groups. These representations also have a nice combinatorial description, using partitions.

In this talk I will discuss the study of representations of $\text{GL}_n(K)$ where $K$ is a finite quotient of a discrete valuation ring (such as $\mathbb{Z}/(p^r)$ or $k[[x]]/(x^r)$, where $k$ is a finite field). One reason to study such representations is that all continuous complex representations of the groups $\text{GL}_n(\mathbb{Z}_p)$ and $\text{GL}_n(k[[x]])$ (where $\mathbb{Z}_p$ denotes the $p$-adic integers) arise from these finite quotients. I will explain why the natural generalization of the Harish-Chandra functors do not furnish a PSH algebra in this case, and how is this related to the Bruhat decomposition and Gauss elimination.

In order to overcome this issue we have constructed a generalization of the Harish-Chandra functors. I will explain this generalization, describe some properties of the new functors and some applications.

The talk will be based on a joint work with Tyrone Crisp and Uri Onn.