Fachseminar *Differentialgeometrie* (Math.)
Seminar *New developments in geometry* (Math.Phys.)

Periodic motions of conservative systems

Winter term 2023/24

Tuesday 11:15 – 12:45, Seminargebäude SG 3-12
Start: Tuesday, October, 10, 2023
Please enrol in **Moodle**

Students:
– mathematics
– mathematical physics (M.Sc.), it is a compulsary elective course in the mathematical physics program (10-MAT-MPHSG resp. 10-MAT-MPDG2)

Topics:
For a conservative system with n degrees of freedom, kinetic energy T and potential function U *brake orbits* are periodic orbits of total energy $T + U = E$ which go through *rest points* in $U^{-1}(E)$. These brake orbits are geodesics of the *Jacobi metric* on $M_E = \{x; U(x) \leq E\}$ and are in close relation with *orthogonal geodesic chords,* i.e. geodesics meeting the boundary $\partial M_E = U^{-1}(E)$ orthogonally. Seifert conjectured that there are n brake orbits if the set M_E is diffeomorphic to an n-disc. For an introduction into this topic see [GZ].
The second topic of the seminar are existence results for closed geodesics and the characterization of a metric on a 2-sphere all of whose geodesics are closed.

List of talks:

I. Seifert's conjecture on the number of brake orbits

1. Introduction and basic facts, [GGP], Section 2, p. 7-11

2. Functional framework, [GGP], Section 3, p.11–15

3. $\mathcal{V}^-$-critical curves, [GGP], Sectionb 4, p. 15–19

4. $\mathcal{V}^+$-vector fields and the invariant set, [GGP], Section 5, p. 19–26

5. Deformation results and the proof of the main result, [GGP], Section 6, p. 26–31

II. Closed geodesics

1. Closed geodesics on connected sums, [RT] Section 2, 3

2. Homology generated by iterated closed geodesics, [BK]

3. Zoll metrics on the 2-sphere and the simple length spectrum, [MS]

4. Closed geodesics on non-compact manifolds, [AM]

References:

[AM] L.Asselle, M.Mazzucchelli, On the existence of inifinitely man closed geodesics on non-compact manifolds, Proc.Amer.Math.Soc. 145 (2017) 2689–2697

[BK] V.Bangert, W.Klingenberg, Homology generated by iterated closed geodesics, topology 22 (1983) 379–388

[GGP] R. Giamboò, F. Giannoni, and P. Piccione, Multiple orthogonal geodesic chords and a proof of Seifert's conjecture on brake orbits, arXiv:2002.09687

[GZ] H. Gluck and W. Ziller, Existence of periodic motions of conservative systems, In: Seminar on minimal submanifolds, Princetion University Press 1983

[MS] M. Mazzucchelli, S. Suhr, A characterization of Zoll riemannian metrics on the 2-sphere, Bull. London Math. Soc. 50 (2018) 997–1006

[RT] H.B.Rademacher, I.Taimanov, Closed geodesics on connected sums and 3-manifolds, J. Differential Geom. 120 (2022) 557 - 573