Folie 5

a) $$\left(\ln(4x^3)\right)' = \frac{1}{4x^3} \cdot 12x^2 = \underline{\underline{\frac{3}{x}}}$$

b) $$\left(\tan(\cos(-2x))\right)' = \frac{1}{\left(\cos(\cos(-2x))\right)^2} \cdot \left(-\sin(-2x)\right) \cdot (-2)$$

$$= \underline{\underline{\frac{2\sin(-2x)}{\left(\cos(\cos(-2x))\right)^2}}}$$

c) $$\left(\frac{\cos(x)}{x^4}\right)' = \left(\cos(x) \cdot x^{-4}\right)' = \left(-\sin(x)\right) \cdot x^{-4} + \cos(x) \cdot (-4) \cdot x^{-5}$$

$$= \frac{-\sin(x)}{x^4} - \frac{4 \cdot \cos(x)}{x^5}$$

$$= \underline{\underline{\frac{1}{x^4}\left(-\sin(x) - \frac{4\cos(x)}{x}\right)}}$$

d) $$\left(x \cdot \sin\left(\frac{1}{x}\right)\right)' = \sin\left(\frac{1}{x}\right) + x \cdot \cos\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right) = \underline{\underline{\sin\left(\frac{1}{x}\right) - \frac{\cos\left(\frac{1}{x}\right)}{x}}}$$

e) $$\left(x^2 \cdot \sin\left(\frac{1}{x}\right)\right)' = 2x \cdot \sin\left(\frac{1}{x}\right) + x^2 \cdot \cos\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right) = \underline{\underline{2x \cdot \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)}}$$

Folie 6

$$f(x) = -6x^3 + 9x^2 \qquad [0,2]$$

$$f'(x) = -18x^2 + 18x$$

$$f''(x) = -36x + 18$$

Nullstellen

$$0 = -6x^3 + 9x^2 \Rightarrow 0 = 3x^2(-2x+3)$$

$$\hookrightarrow x_1 = 0 \qquad \hookrightarrow x_2 = \frac{3}{2}$$

lokale Max./Min

$$0 = -18x^2 + 18x \Rightarrow 0 = 18x \cdot (-x+1)$$

$$\hookrightarrow x_1 = 0 \qquad \hookrightarrow x_2 = 1$$

$$f''(0) = 18 > 0 \rightarrow x_{min} = 0$$

$$f''(1) = -18 < 0 \rightarrow x_{max} = 1$$

$f(0) = 0 \rightarrow T(0|0)$ lokales Minimum

$f(1) = 3 \rightarrow H(1|3)$ lokales Maximum

Randpunkte überprüfen: $f(0) = 0$

$$f(2) = -6 \cdot 8 + 9 \cdot 4 = -12$$

$\Rightarrow$ globales Minimum bei $(2|-12)$

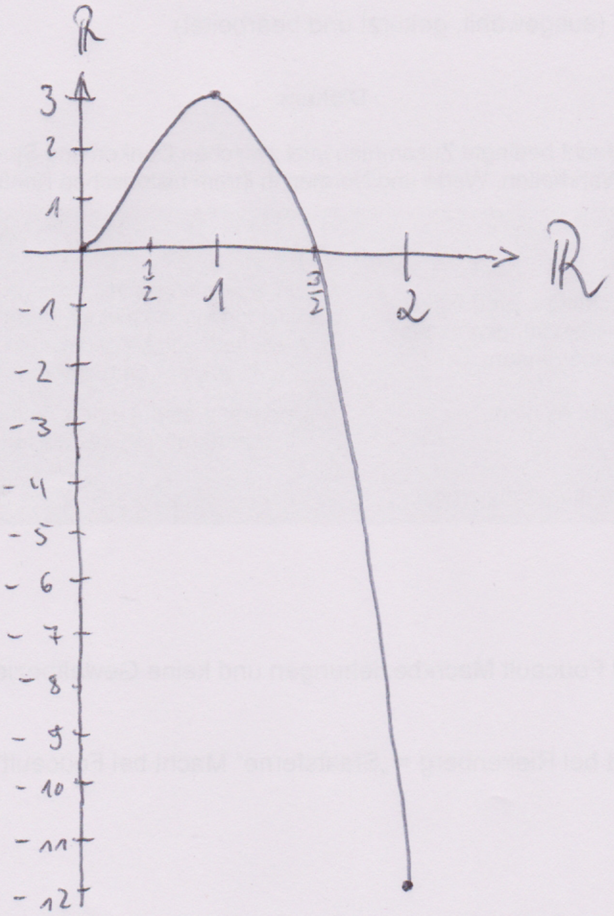
$\Rightarrow H(1|3)$ ist auch globales Maximum

Verhalten im Unendlichen

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} -6x^3 + 9x^2 = \lim_{x \to -\infty} -x^3 \underbrace{\left(6 - \frac{9}{x}\right)}_{=6} = \underline{+\infty}$$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} -x^3 \left(6 - \frac{9}{x^2}\right) = \underline{-\infty}$$

Skizze:



Folie 8

a) $$\lim_{x \to 0} x \cdot \ln(x) = \lim_{x \to 0} \frac{\ln(x)}{\frac{1}{x}} = „-\frac{\infty}{\infty}“$$

l'Hospital

$$\Longrightarrow \lim_{x \to 0} \frac{\frac{1}{x}}{\left(-\left(\frac{1}{x}\right)^2\right)} = \lim_{x \to 0} \frac{1}{-\left(\frac{1}{x}\right)} = \lim_{x \to 0} -x = 0$$

$$\Longrightarrow \lim_{x \to 0} x \cdot \ln(x) = \underline{0}$$

f) $\lim\limits_{x \to \infty} \frac{x}{e^x} = „\frac{\infty}{\infty}“$

$\overset{\text{l'Hospital}}{\Longrightarrow} \lim\limits_{x \to \infty} \frac{1}{e^x} = „\frac{1}{\infty}“ = 0 \Rightarrow \lim\limits_{x \to \infty} \frac{x}{e^x} = \underline{\underline{0}}$

c) $\lim\limits_{x \to 0} \frac{e^x - 1}{\sin(x)} = „\frac{0}{0}“ \overset{\text{l'Hospital}}{\Longrightarrow} \lim\limits_{x \to 0} \frac{e^x}{\cos(x)} = \frac{1}{1} = 1$

$\Longrightarrow \lim\limits_{x \to 0} \frac{e^x - 1}{\sin(x)} = \underline{\underline{1}}$

<u>Folie 9</u>

a) $\frac{\partial f}{\partial x} = e^{\sin(x \cdot y)} \cdot \cos(x \cdot y) \cdot y + e^{\cos(x+y)} \cdot (-\sin(x+y)) \cdot 1$

$\frac{\partial f}{\partial y} = e^{\sin(x \cdot y)} \cdot \cos(x \cdot y) \cdot x + e^{\cos(x+y)} \cdot (-\sin(x+y))$

b) $\frac{\partial f}{\partial x} = \ln(x \cdot y) + x \cdot \frac{1}{x \cdot y} \cdot y = \underline{\ln(x \cdot y) + 1}$

$\frac{\partial f}{\partial y} = x \cdot \frac{1}{x \cdot y} \cdot x = \underline{\underline{\frac{x}{y}}}$

Folie 10

a) $$\int_0^1 \underset{f'}{e^x}\cdot\underset{g}{(2-x^2)}\,dx = \underbrace{\left[e^x\cdot(2-x^2)\right]_0^1}_{=e\cdot(2-1)-(1\cdot 2)} - \int_0^1 e^x\cdot(-2x)\,dx = e-2+\int_0^1 \underset{f'}{e^x}\cdot\underset{g}{2x}\,dx$$

$$= e-2+\left[e^x\cdot 2x\right]_0^1 - \int_0^1 e^x\cdot 2\,dx = e-2+2e-0-\left[2e^x\right]_0^1$$

$$= 3e-2-(2e-2) = \underline{\underline{e}}$$

b) $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \underset{f}{x^2}\cdot\underset{g'}{\sin(x)}\,dx = \left[x^2\cdot(-\cos(x))\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2x\cdot(-\cos(x))\,dx$$

$$= \frac{\pi^2}{4}\cdot 0 - \frac{\pi^2}{4}\cdot 0 + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \underset{f}{2x}\cdot\underset{g'}{\cos(x)}\,dx$$

$$= \left[2x\cdot\sin(x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cdot\sin(x)\,dx$$

$$= \underbrace{\pi\cdot 1-(-\pi\cdot(-1))}_{=0} - 2\cdot\underbrace{\left[-\cos(x)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}_{=0} = \underline{\underline{0}}$$

c) $$\int_{-1}^{1} (2x-3)^4\,dx$$

<u>Substitution</u> $y = 2x-3$

<u>Ableitung</u> $\frac{dy}{dx} = 2 \Rightarrow dx = \frac{dy}{2}$

<u>Grenzen:</u> aus -1 wird $-2-3=-5$

aus 1 wird $2-3=-1$

$$\int_{-1}^{1} (2x-3)^4\,dx = \int_{-5}^{-1} y^4\,\frac{dy}{2} = \frac{1}{2}\left[\frac{1}{5}y^5\right]_{-5}^{-1} = \frac{1}{2}\cdot\left(\frac{1}{5}\cdot(-1)^5-(-5)^5\cdot\frac{1}{5}\right)$$

$$= -\frac{1}{10}+\underline{\frac{625}{2}} = -0{,}1+312{,}5$$

$$= \underline{\underline{312{,}4}}$$

d) $\int_{-1}^{1} \frac{x^2}{\sqrt{1-x^3}}\, dx$

$\boxed{\begin{array}{l} \underline{\text{Substitution}}: w = 1-x^3 \\ \underline{\text{Ableitung}}: \frac{dw}{dx} = -3x^2 \Rightarrow dx = -\frac{dw}{3x^2} \\ \underline{\text{Grenzen}}: \text{aus } -1 \text{ wird } 1-(-1) = 2 \\ \qquad\qquad\; \text{aus } 1 \text{ wird } 1-1 = 0 \end{array}}$

$$\int_{-1}^{1} \frac{x^2}{\sqrt{1-x^3}}\, dx = -\int_{2}^{0} \frac{\cancel{x^2}}{\sqrt{w}} \frac{dw}{3\cancel{x^2}} = \frac{1}{3}\int_{0}^{2} \frac{1}{\sqrt{w}}\, dw = \frac{1}{3}\left[2\cdot\sqrt{w}\right]_0^2 = \underline{\underline{\frac{2\cdot\sqrt{2}}{3}}}.$$