1. Berechnen Sie

a) $(4+8i)\cdot(3-2i)^2 = (4+8i)\cdot(9-12i-4) = (4+8i)\cdot(5-12i) = (20-8i+96) = \underline{\underline{116-8i}}$

b) $\dfrac{4+8i}{(3+2i)^2} = \dfrac{4+8i}{5+12i} = \dfrac{(4+8i)\cdot(5-12i)}{5^2+12^2} = \dfrac{20-8i+96}{169} = \underline{\underline{\dfrac{116}{169} - \dfrac{8}{169}i}}$

c) $\left|\dfrac{17-i}{1+i}\right| = \left|\dfrac{17-18i-1}{2}\right| = |8-9i| = \sqrt{64+81} = \underline{\underline{\sqrt{145}}}$

d) $\left|\dfrac{1}{3+7i}\right| = \left|\dfrac{3-7i}{9+49}\right| = \dfrac{1}{58}\cdot|3-7i| = \underline{\underline{\dfrac{\sqrt{58}}{58}}}$

e) $\left|\left(\dfrac{1+i}{1-i}\right)^2\right| = \left|\left(\dfrac{(1+i)\cdot(1+i)}{2}\right)^2\right| = \left|\left(\dfrac{2i}{2}\right)^2\right| = |i^2| = |-1| = \underline{\underline{1}}$

f) $\left|\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^3\right| = \left|\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\cdot\left(\frac{1}{4}-\frac{\sqrt{3}}{2}i-\frac{3}{4}\right)\right| = \left|\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\cdot\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)\right| = \left|\left(\frac{1}{4}-\frac{3}{4}i^2\right)\right| = \underline{1}$

g) $\dfrac{7+5i}{i-1} = \dfrac{(7+5i)\cdot(-1-i)}{2} = \dfrac{-2-12i}{2} = \underline{\underline{-1-6i}}$

h) $\dfrac{5\cdot(i+1)}{3-4i} + \dfrac{20}{4+3i} = \dfrac{(5i+5)\cdot(3+4i)+20\cdot(4-3i)}{25} = \dfrac{35i-5+80-60i}{25} = \underline{\underline{3-i}}$

i) $\left|\dfrac{2\cdot\sqrt{6}-i}{2+3i}\right| = \left|\dfrac{4\cdot\sqrt{6}-3-(2i+6\cdot\sqrt{6}i)}{13}\right| = \dfrac{1}{13}\cdot\sqrt{(4\cdot\sqrt{6}-3-(2i+6\cdot\sqrt{6}i))\cdot(4\cdot\sqrt{6}-3+2i+6\sqrt{6}i)}$

$= \dfrac{1}{13}\cdot\sqrt{96+9+4+216} = \dfrac{\sqrt{325}}{13} = \sqrt{\dfrac{25}{13}} = \underline{\underline{\dfrac{5\cdot\sqrt{13}}{13}}}$

2.) Wann gilt für $a,b \in \mathbb{C}: |a+b| = |a|+|b|$ ?

Es gilt: $|a+b| \leqq |a|+|b|$

$$\Leftrightarrow \sqrt{a\bar{a}+a\bar{b}+\bar{a}b+b\bar{b}} \leqq \sqrt{a\bar{a}}+\sqrt{b\bar{b}} \quad \mid \text{Quadrieren (möglich, da alle Ausdrücke} > 0)$$

$$\Leftrightarrow a\bar{a}+a\bar{b}+\bar{a}b+b\bar{b} \leqq a\bar{a}+2\cdot\underbrace{\sqrt{a\bar{a}b\bar{b}}}_{=|a\cdot b|}+b\bar{b}$$

$$a\bar{b}+\bar{a}b \leqq 2\cdot|ab| = 2\cdot|a\bar{b}| \qquad \text{Setze } z := a\bar{b} = u+i\cdot v$$

$$z+\bar{z} \leqq 2\cdot|z|$$

$$2u \leqq 2\cdot\sqrt{u^2+v^2} \Rightarrow \text{Gleichheit bei } v=0$$

$$\underline{\underline{a\bar{b} = u \in \mathbb{R}}}$$

4.) Berechnen Sie:

a) $\sqrt[6]{729} \Rightarrow \varphi = 0 \qquad \sqrt[6]{729} = \underbrace{\sqrt[6]{|729|}}_{=3}\cdot e^{\frac{1}{6}\cdot i\cdot(0+2\cdot k\cdot\pi)} \quad , k = 0,1,2,3,4,5$

$$k=0:\ 3\cdot e^{\frac{1}{6}\cdot i\cdot 0} = \underline{\underline{3}}$$

$$k=1:\ 3\cdot e^{\frac{1}{6}\cdot i\cdot 2\pi} = 3\cdot e^{\frac{1}{3}\pi i} = 3\cdot\Big(\underbrace{\cos\big(\tfrac{\pi}{3}\big)}_{=\frac{1}{2}}+i\cdot\underbrace{\sin\big(\tfrac{\pi}{3}\big)}_{=\frac{\sqrt{3}}{2}}\Big) = \underline{\underline{\frac{3}{2}+i\cdot\frac{3\cdot\sqrt{3}}{2}}}$$

$$k=2:\ 3\cdot e^{\frac{2}{3}\pi\cdot i} = 3\cdot\Big(\underbrace{\cos\big(\tfrac{2\pi}{3}\big)}_{=-\frac{1}{2}}+i\cdot\underbrace{\sin\big(\tfrac{2\pi}{3}\big)}_{=\frac{\sqrt{3}}{2}}\Big) = \underline{\underline{-\frac{3}{2}+i\cdot\frac{3\cdot\sqrt{3}}{2}}}$$

$$k=3:\ 3\cdot e^{\pi\cdot i} = 3\cdot(\cos(\pi)+i\cdot\sin(\pi)) = \underline{\underline{-3}}$$

$$k=4:\ 3\cdot e^{\frac{4\pi}{3}\cdot i} = 3\cdot\Big(\cos\big(\tfrac{4\pi}{3}\big)+i\cdot\sin\big(\tfrac{4\pi}{3}\big)\Big) = \underline{\underline{-\frac{3}{2}-i\cdot\frac{3\cdot\sqrt{3}}{2}}}$$

$$k=5:\ 3\cdot e^{\frac{5\pi}{3}i} = 3\cdot\Big(\cos\big(\tfrac{5\pi}{3}\big)+i\cdot\sin\big(\tfrac{5\pi}{3}\big)\Big) = \underline{\underline{\frac{3}{2}-i\cdot\frac{3\sqrt{3}}{2}}}$$

Zeichnung von 4a)