

$$2. (a) \int_{|z|=r} \frac{dz}{(z-a)^n (z-b)^m} = \int_{|z|=r} \frac{f(z)}{(z-a)^n} dz \quad \text{mit } f(z) = \frac{1}{(z-b)^m}$$

verallg. \rightarrow
Cauchy-Int. Formel

$$= \frac{2\pi i}{(n-1)!} \cdot f^{(n-1)}(a)$$

$$= \frac{2\pi i}{(n-1)!} \cdot (-1)^{n-1} \cdot \frac{m \cdot (m+1) \cdot \dots \cdot (m+n-2)}{(a-b)^{m+n-1}}$$

$$= \binom{m+n-2}{n-1} \cdot \frac{(-1)^{n-1} \cdot 2\pi i}{(a-b)^{m+n-1}}$$

$$(b) f(z) = \frac{z^2 + (1-2i)z - 2}{z^3 - (1+2i)z^2 + (2i-1)z + 1} = \frac{z^2 + (1-2i)z - 2}{(z-1)(z-i)^2}$$

PBZ... $\rightarrow = \frac{1}{z-1} + \frac{1}{(z-i)^2}$

$$r < 1: \int_{|z|=r} f(z) dz = 0$$

$$r > 1: \int_{|z|=r} f(z) dz = \underbrace{\int_{|z|=r} \frac{1}{z-1} dz}_{= 2\pi i} + \underbrace{\int_{|z|=r} \frac{1}{(z-i)^2} dz}_{= 0}$$

$$= 2\pi i$$
