# Exercise Sheet 12

Discussion on 02.02.24

This exercise sheet is thought as guideline for a repetition of the lecture. These kinds of questions could be asked in an oral examination.

**Exercise 1**

1. What is the idea behind Runge-Kutta methods?
2. Do the (discrete) solutions of implicit Runge-Kutta methods exist
   (a) for all $\tau$?
   (b) for all $\tau$ that are small enough?
3. Are multistep-methods stable for all $\tau$ that are small enough? Are there other conditions for stability?
4. What does $A$ stability mean? Are there any explicit Runge-Kutta methods that are $A$ stable?

**Exercise 2**

1. Are the following statements correct?
   a) An explicit approximation of a time-dependent PDE has finite speed of propagation.
   b) An implicit approximation of a time-dependent PDE has infinite speed of propagation.
2. What are the advantages and disadvantages of explicit methods?

**Exercise 3**

1. Which rate of convergence does the $P_k$-FEM have?
2. Does $\|u - u_h\|_{L^2(\Omega)} \lesssim \|u - v_h\|_{L^2(\Omega)}$ hold for the Galerkin approximation $u_h \in V_h$ of the Poisson-problem and arbitrary $v_h \in V_h$?
3. How does the weak formulation of the PDE

$$-\operatorname{div} A\nabla u + cu = f$$

look like? Under which conditions exists a solution? What does a finite element approximation look like? Which rate of convergence does this FEM have?