

Exercise Sheet 5

Discussion on 24.11.23

Exercise 1

Show that the stability function of an implicit Runge-Kutta method is a rational function.

Exercise 2

Examine the Radau-3-method for A-stability. The method is given by the Butcher-tableau

$$\begin{array}{c|cc} 1/3 & 5/12 & -1/12 \\ 1 & 3/4 & 1/4 \\ \hline & 3/4 & 1/4 \end{array}.$$

Exercise 3

- (a) Show the following fact: *If G is convex then G is μ -convex for each $\mu > 0$.*
- (b) Construct a μ -convex function which is not convex.

Exercise 4

Implement the Newton method to solve the (possibly) non linear system arising from the implicit Euler scheme. Apply the explicit and the implicit Euler schemes to the ODEs $y'(t) = \lambda y(t)$ and $y'(t) = -G'(y(t))$ with the function $G(s) = (s - 1)^2 (s + 1)^2$ with initial values $y_0 \in \{-2, -0.5, 0, 0.5, 2\}$.