Exercise Sheet 3

Discussion on 10.11.23

Exercise 1
Let $(\hat{\alpha}_\ell, \hat{\beta}_\ell)_{\ell=0,\dots,m}$ define a linear, explicit multistep method and $(\alpha_\ell, \beta_\ell)_{\ell=0,\dots,m}$ define a linear, implicit multistep method. The approximation y_{k+m} is defined by $y_{k+m} = y_{k+m}^{(j)}$, where $y_{k+m}^{(j)}$ is defined by the following algorithm:

1. Compute the solution of the explicit method, i.e., compute
$$\tilde{y}_{k+m} = -\sum_{\ell=0}^{m-1} \hat{\alpha}_\ell y_{k+\ell} + \tau \sum_{\ell=0}^{m-1} \hat{\beta}_\ell f(t_{k+\ell}, y_{k+\ell}).$$
2. Set $y_{k+m}^{(0)} = \tilde{y}_{k+m}$.
3. Compute j steps of the fixpoint iteration of the implicit method, i.e., compute $y_{k+m}^{(j)}$ with
$$y_{k+m}^{(i+1)} = -\sum_{\ell=0}^{m-1} \alpha_\ell y_{k+\ell} + \tau \sum_{\ell=0}^{m-1} \beta_\ell f(t_{k+\ell}, y_{k+\ell}) + \tau \beta_m f(t_{k+m}, y_{k+m}^{(i)}).$$

Show that, if f is Lipschitz continuous in the second argument, this algorithm defines an explicit multistep method with order of consistency $p = \min\{p_{\text{expl}} + j, p_{\text{impl}}\}$, where p_{expl} und p_{impl} are the orders of the explicit resp. the implicit method.

Exercise 2
Determine the coefficients of the Adams-Bashforth-method and the Adams-Moulton-method for $m = 3$.

Exercise 3
Show that the Adams-Bashforth-method with m steps is consistent of order m and the Adams-Moulton-method with m steps is consistent of order $m + 1$.

Exercise 4 (Programming exercise)
Implement the Adams-Bashforth-method with 3 steps and compare the results to those of the already implemented methods.