Exercise Sheet 2

Discussion on 03.11.23

Exercise 1
Check the conditions of Theorem 3.6 from the lecture for the following Runge-Kutta methods:

- Euler-Collatz method
- implicit trapezoidal method
- Heun method
- classical Runge-Kutta method

Exercise 2
Check the conditions of Theorem 3.7 from the lecture for the following Runge-Kutta methods:

- Euler-Collatz method
- implicit trapezoidal method
- Heun method
- classical Runge-Kutta method

Exercise 3
If Φ is Lipschitz continuous in the third entry, one can use

$$\tilde{C}(t_k, z_k, \tau) := \frac{z(t_{k+1}) - z_k}{\tau} - \Phi(t_k, z_k, z(t_{k+1}), \tau)$$

instead of $C(t_k, z_k, \tau)$ to determine the order of consistency.

Exercise 4 (Programming exercise)

1. Implement the Euler-Collatz method and the implicit trapezoidal method. You can use the function `fixpoint_iteration` from `ODE_solver.jl` to solve the non-linear system of equations.
2. Implement a routine that computes the ℓ^∞ error of $(y_k)_{k=1,\dots,K}$ and $(y(t_k))_{k=1,\dots,K}$, if the exact solution is known.
3. Compare the methods to the explicit Euler-method, the implicit Euler-method and the implicit midpoint method.