EXERCISES 6.2 (submit by 22.05.2015)

1. Write the Taylor polynomial of degree 2 near $(0,0)$ for the following functions:
   (a) $f(x,y) = e^{x+y^2}$,
   (b) $f(x,y) = \sin(x-y)$.
2. Prove that the ball $B(x,r) = \{y \in \mathbb{R}^n \ : \ \|y-x\| < r\}$ is convex.
3. Let $y = f(x_1,\ldots,x_n)$ be differentiable on $B(0,r)$. Assume that $f_{x_i}(x) = 0$ for all $x \in B(0,r)$ and $i \in \{1,\ldots,n\}$. Prove that $f(x) = 0$ for all $x \in B(0,r)$. [Hint: use Lagrange's mean value theorem.]
4. Find local extrema of the following functions:
   (a) $z = x^2 + y^3$,
   (b) $z = x^2 + y^4$.
5. Find the maximum and minimum of the function $z = x^2 + y^2 - 2x + 1$ on the rectangle $D = \{(x,y) \ : \ 0 \leq x \leq 4, \ 0 \leq y \leq 2\}$.