EXERCISES 5.1 (submit by 15.05.2015)

1. Which of the following limits exist?
   (a) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$
   (b) $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$
   (c) $\lim_{(x,y)\to(0,0)} \frac{x-y+x^2+y^2}{x+y}$ (here the domain of the function is $\{(x,y) \,:\, x+y>0\}$)
   (d) $\lim_{(x,y)\to(0,0)} x\sin\frac{1}{y}$ (here the domain of the function is $\{(x,y) \,:\, y\neq 0\}$)
   (e) $\lim_{(x,y)\to(0,1)} \frac{e^x-y}{xy}$ (here the domain of the function is $\{(x,y) \,:\, xy>0\}$)

2. Prove that the following function is continuous on $\mathbb{R}^2$:

$$f(x,y)=\begin{cases} \frac{x^2y}{x^2+y^2} & \text{if } x^2+y^2>0,\\ 0 & \text{if } x=y=0.\end{cases}$$

3. Compute the limit

$$\lim_{(x,y,z)\to(0,0,0)} \frac{\sin(x^2+y^2+z^2)}{x^2+y^2+z^2}.$$