EXERCISES 2.2 (submit by 24.04.2015)

In all the exercises, V is a finite-dimensional inner product space over $\mathbb{C}$.

1. Linear operator T is called *self-adjoint* if $T^* = T$. Prove the following properties of self-adjoint operators:
   (a) If T_1 and T_2 are self-adjoint, then T_1^2, T_1+T_2, αT_1 (here $\alpha \in \mathbb{C}$), and $T_1T_2+T_2T_1$ are self-adjoint. (Note that T_1T_2 is not necessarily self-adjoint!)
   (b) If T is self-adjoint, then all its eigenvalues are real numbers.
   (c) T is self-adjoint if and only if there exists an orthonormal basis in which the matrix of T is diagonal with real entries.

2. Linear operator T is called *skew-adjoint* if $T^* = -T$. Prove the following properties of skew-adjoint operators:
   (a) If T is skew-adjoint, then all its eigenvalues are purely imaginary.
   (b) T is skew-adjoint if and only if there exists an orthonormal basis in which the matrix of T is diagonal with purely imaginary entries.
   (c) T is skew-adjoint if and only if iT is self-adjoint.