EXERCISES 11.1 (submit by 26.06.2015)

- 1. Prove that ℓ_2 is complete.
- 2. Let f be a continuous function from a metric space (X, ρ) to \mathbb{R} . Prove that for any a < b, the set $f^{-1}((a, b)) = \{x \in X : f(x) \in (a, b)\}$ is open.
- 3. Let S be a compact in a metric space (X, ρ) . Let f be a continuous function from X to \mathbb{R} . Prove that $f(S) = \{f(x) : x \in X\}$ is a compact in \mathbb{R} .
- 4. Prove that the norms $\|\cdot\|_1$, $\|\cdot\|_2$, and $\|\cdot\|_\infty$ are equivalent in \mathbb{R}^d .
- 5. Prove that the set of all continuous functions on [0,1] equipped with the norm $\int_0^1 |f(x)| dx$ is not a Banach space.
- 6. Prove that ℓ_{∞} is not a Hilbert space.