EXERCISES 9.1 (submit by 08.06.2016)

1. Let p and q be positive real numbers. Use the second derivative test to find (if any) the maximum and minimum of the following functions:
   (a) $z = \frac{x^2}{2p} + \frac{y^2}{2q}$ (elliptic paraboloid),
   (b) $z = \frac{x^2}{2p} - \frac{y^2}{2q}$ (hyperbolic paraboloid).
2. Use the method of Lagrange multipliers to find the maximum and minimum of the following functions:
   (a) $z = \frac{x}{a} + \frac{y}{b}$ on the circle $x^2 + y^2 = 1$,
   (b) $z = x^2 + y^2$ on the line $\frac{x}{a} + \frac{y}{b} = 1$.
3. Derive the formula for the distance between a point $(x_0, y_0, z_0) \in \mathbb{R}^3$ and a plane $Ax + By + Cz + D = 0$. [Hint: Use Lagrange multipliers to minimize over points (x, y, z) in the plane the square of the distance between (x_0, y_0, z_0) and (x, y, z).]
4. Among all rectangular boxes inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, find the one with the largest volume.
5. Among all triangles inscribed in the unit circle, find the one with the largest area. [Hint: Denote by 0 the center of the circle and by A, B, C the vertices of an inscribed triangle. Let $x = \angle AOB$, $y = \angle BOC$, $z = \angle COA$. Then $x + y + z = 2\pi$, and the area of the triangle equals $\frac{1}{2}\sin x + \frac{1}{2}\sin y + \frac{1}{2}\sin z$.]
6. Let $f(x_1, \ldots, x_n) = \sum_{i,j=1}^{n} a_{ij} x_i x_j$ with $a_{ij} = a_{ji}$ for all i, j. Find the maximum and minimum of f on the set $C = \{(x_1, \ldots, x_n) : x_1^2 + \ldots + x_n^2 = 1\}$.