EXERCISES 8.1 (submit by 03.06.2016)

1. Consider the function

$$f(x,y) = \begin{cases} (x^2 + y^2) \sin \frac{1}{x^2 + y^2} & \text{if } x^2 + y^2 > 0, \\ 0 & \text{if } x = y = 0. \end{cases}$$

Prove that f is differentiable at 0. Notice that f_x and f_y are not continuous at 0.

- 2. Let $f(x, y, z) = xy^2 z^3$. For the direction $\ell = \left(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$, compute $\frac{\partial f}{\partial \ell}(1, 1, 1)$.
- 3. Let $z = x^2y + 3xy$, where $x = \cos t$ and $y = \sin t$. Find $\frac{dz}{dt}$.
- 4. Let $z = e^x \sin y$, where x = st and $y = \frac{s}{t}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.
- 5. Consider the function

$$f(x,y) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2} & \text{if } x^2+y^2 > 0, \\ 0 & \text{if } x = y = 0. \end{cases}$$

Compute f_{xy} and f_{yx} . Observe that they are not equal.

- 6. Write the Taylor polynomial of degree 2 near (0,0) for the following functions:
 - (a) $f(x, y) = e^{x+y^2}$.
 - (b) $f(x, y) = \sin(x y)$.
- 7. Find local extrema of the following functions:
 - (a) $z = x^2 + y^3$, (b) $z = x^2 + y^4$.
- 8. Find the maximum and minimum of the function $z = x^2 + y^2 2x + 1$ on the rectangle $D = \{(x, y) : 0 \le x \le 4, 0 \le y \le 2\}.$