

EXERCISES 7.1 (submit by 25.05.2016)

1. Which of the following limits exist?

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + y^3}{x^2 + y^2}$

(c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x - y + x^2 + y^2}{x + y}$ (here the domain of the function is $\{(x, y) : x + y > 0\}$)

(d) $\lim_{(x,y) \rightarrow (0,0)} x \sin \frac{1}{y}$ (here the domain of the function is $\{(x, y) : y \neq 0\}$)

(e) $\lim_{(x,y) \rightarrow (0,1)} \frac{e^x - y}{xy}$ (here the domain of the function is $\{(x, y) : xy > 0\}$)

2. Prove that the following function is continuous on $\mathbb{R}^2$:

$$f(x, y) = \begin{cases} \frac{\sin(x^2 y)}{x^2 + y^2} & \text{if } x^2 + y^2 > 0, \\ 0 & \text{if } x = y = 0. \end{cases}$$

3. Compute the limit

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{\sin(x^2 + y^2 + z^2)}{x^2 + y^2 + z^2}.$$